A New Short
TREATISE
OF
Algebra:

With the Geometrical Conſtruction of
EQUATIONS,
As far as
The Fourth *Power* or *Dimenſion*.
Together with a Specimen of the
NATURE and ALGORITHM
OF
FLUXIONS.

The Second Edition, with Additions.

By *JOHN HARRIS*, M. A. and Fellow of the *Royal Society*.

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BRITISH MUSEUM

THE
Epiſtle Dedicatory
TO THE
Learned and Ingenious
JOHN BRIDGES **Eſq;**
Sollicitor of the Cuſtoms.

Dear SIR,

WHEN I conſider the ancient Date of our Friendſhip, and reflect, as I always do with Pleaſure, on the many happy Hours we have ſpent together, and that too in Diſcourſes we need not now be aſham'd of: When I ſee ſtill in my Friend the ſame true *Engliſh* Honeſty and Integrity, the ſame ſincere Love for Virtue and Honour, and the ſame warm Affection for all uſeful and ſubſtantial Learning, which at firſt made me juſtly Admire him and Devote my ſelf to him: When I conſider a Genius every way improv'd (and no way injur'd) by Travels into Countries, from whence too many bring home nothing but Vice and Impertinence: And when I am ſo happy as to find by Experience, that even the hurry of Buſineſs it ſelf cannot make Mr. *Bridges* forget or neglect either Learning or his Friend: Then would I gladly ſhew that I am neither Unjuſt nor Ungrateful, but as doubly a Debtor both to your Merit and to your Friendſhip, tell the World I have a due Senſe of both: Permit me then, *Dear Sir!* to do it this way, and to continue, as I have long done,

Your Real Friend,

and moſt Obliged Humble Servant,

John Harris.

TO THE
READER.

THis ſmall Tract of that Admirable Science, Algebra, *was written primarily for the Uſe of my Auditors at the* Publick Mathematick Lecture; *which was ſet up at the* Marine Coffee-houſe *in* Birchin-Lane, *entirely for the Publick Good, by the Generous* Charles Cox *Eſq; Member of Parliament for the Burgh of* Southwark.

The Book is ſhort indeed, but I think plain enough every where, eſpecially in thoſe Parts which have been leſs Treated of in our own Language, viz. The Geometrical Conſtruction of Equations: *Which I have carried as far as to* Biquadraticks, *and have ſhew'd you plainly how to Conſtruct all Equations not exceeding four Dimenſions, by the help of a Circle interſecting the Curve of a* Parabola.

I have alſo demonſtrated the Properties *of the* Parabola *on which thoſe Conſtructions depend;*

depend; and have given you beſides, the Method of the Inveſtigation of the famous Mr. Baker's Central Rule.

N. B. Mr. *Hayes's* Book was not then Publiſhed.

And becauſe I have not yet found it done by any one in the Engliſh *Tongue, I have given you a ſhort Account of the Nature and Algorithm of* Fluxions, *and one or two Inſtances of their Uſe and Application: But in this I have been deſignedly as brief as poſſible I could, intending by it only to ſtir up the Readers Curioſity to peruſe thoſe Excellent Treatiſes which I have there mentioned: And which when he hath done, I know I ſhall have his Thanks for that little* Sketch *of* Fluxions, *which he will find there.*

In the whole I have propoſed to my ſelf rather to Inſtruct *the Young* Engliſh *Beginner, than to* improve *the Learned Proficient; as knowing that if I can but help to lay a good ſubſtantial foundation for him, his own Diligence and Application will raiſe the Structure to what height he pleaſes; but without beginning right (which it* *is too commonly neglected) nothing is to be done.*

In this Second Edition, *I have endeavoured to Correct the Fault's of the former; and have added ſome few things in the* Fluxions, *to make that Matter as plain and intelligible as I could, in ſo little a room.*

And to thoſe who would purſue this Matter farther, I recommend Mr. Hayes's *Treatiſe of* Fluxions, *Publiſhed ſince the* Firſt Edition *of the* Compendium.

ALGEBRA.

INTRODUCTION.

1. THE Name *Algebra*, Dr. *Wallis* acquaints us, is Derived from the two firſt Words of *Al-giabr*, *Wolmokabala*, which in the *Arabick* Tongue ſignifies; *The Art of Reſtitution and Compariſon*, or *The Art of Reſolution and Equation*. It was unqueſtionably known to the Ancient *Grecians* (for there are plain Footſteps of it in *Theon* upon *Euclid*, in *Pappus*, and eſpecially in *Archimedes* and *Apollonius*) but it was ſtudiouſly conceal'd by them, and kept as a great Secret. It was yet of more Ancient Uſe among the *Arabians*, who are ſuppoſed to have received it from the *Perſians*, and they from the *Indians*. From the *Arabs*, the *Moors* and *Saracens* brought it into *Spain*: From whence it came into *England*: As did alſo the Uſe of the Numeral Figures, Mathematicks in general, and Aſtronomy in particular, much about the

ſame Time. The firſt *European* Writer of *Algebra* was one *Lucas de Burgo*, or *Lucas Pacciolus*: His Book was Printed at *Venice* in *Italian* A. D. 1494. a good while before we knew any thing of *Diophantus*. This *Great Art*, as *Lucas de Burgo* and *Cardan* call it, may be defined, or rather deſcrib'd, to be *an Analytical Way of Demonſtration, where, aſſuming the Quantity ſought as if it were known and granted; by the help of one or more Quantities really given or known, we proceed by Conſequences, till at laſt the Quantity firſt ſought, and only ſuppoſed to be known, is found equal to ſome real known Quantity, and ſo is it ſelf (of Conſequence) diſcovered.*

2. The Quantity thus ſought is called the *Root*, which being unknown cannot be really Expreſs'd; but may be deſign'd by any Symbol or Character at pleaſure. I (with moſt others) uſe *Vowels* for unknown, and *Conſonants* for known, or given Quantities; as (*a*) or (*e*) for a Root ſought. Tho' *Des Cartes* and his Followers and moſt Foreign Writers uſe the laſt Letters of the Alphabet *x*, *y*, and *z*, for unknown Quantities, and the former Letters as *a*, *b*, *c*, *d*, *&c.* for known ones.

3. The Art of *Algebra* doth much depend on the knowledge of ſome certain Quantities, by the Ancients called *Coſſick* Quantities, but moſt uſually Powers. Which Quantities ariſe from a Rank of Numbers in continual Proportion Geometrical, beginning from Unity: for every Term, but the firſt (or Unity) is call'd ſome *Power*, as in theſe, 1, 2, 4, 8, 16, 32, *&c.* The firſt Term (2) is

the *Root* or firſt Power; the ſecond Term (4) is the *Square* or ſecond *Power*. The third Term (8) is the Cube or third Power, *&c.*

So in a Rank of Fractions deſcending from Unity in the ſame Proportion: as 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, *&c.* the ſeveral Terms have alſo the ſame Names of *Powers*.

4. Hence 'tis clear that any Number being taken as a Root, the ſecond *Power* or *Square* will be produc'd by Multiplying the Root by it ſelf; the third *Power* or *Cube*, by Multiplying the ſecond Power by the *Root*, &c.

5. If over ſuch a Rank of Numbers in Geometrical Proportion continued, there be placed a Series of Numbers beginning with (1) and proceeding orderly, in an Arithmetical Progreſſion, as 1, 2, 3, 4, 5, 6, *&c.* thoſe Numbers are properly called *Indexes* or *Exponents*: Becauſe they both ſhew the Order and Place of each Power; and alſo its *Dimenſions*: (*i. e.*) how often the Root is Multiplyed into it ſelf to produce that Power: For as many Unites as are in the *Exponent*, ſo many Powers is that Number from Unity (*v. g.*) if the Index be 5, the Power under it, is the fifth Power, or the *Biquadrate* multiplyed by the Root.

6. The Sum ariſing from the Addition of any two Indexes makes another, ſhewing what *Power* would be produced by the Multiplication of the two *Powers* Anſwering: So, that by adding them, Multiplication is made in the Numbers themſelves,

on which depends the Nature and Uſe of the Logarithms: And ſo on the contrary, Diviſion in the Correſponding Numbers Anſwers to the Subtraction of the Indexes one from another.

7. Powers from any Letter repreſenting the Root are produced, by repeating it ſo often as the Index of the Powers requires; ſo from the Root (*a*) the Square is (*aa*) the Cube (*aaa*) the Biquadrate (*aaaa*) the fifth Power (*aaaaa*) the ſixth (*aaaaaa*) the ſeventh (*aaaaaaa*) &c. or as *Des Cartes* by the Indexes chuſes rather to expreſs it, a^2, a^3, a^4, a^5, a^6, a^7, &c. which is ſhorter and more convenient in many Caſes.

8. In a Rank of Fractional Numbers deſcending from Unity (as before) the Indexes are all Negative, and are imagined to have this Sign (—) before them, which is implyed in Writing them Fraction-wiſe, thus $\bar{1}$, $\bar{2}$, $\bar{3}$, $\bar{4}$, $\bar{5}$, &c.

9. *Like Quantities* in Algebra are ſuch as are Expreſs'd by the ſame Letters equally repeated in each Quantity; as *a* and *a*, *b* and *b*, *b c d* and *b c d*. *Unlike Quantities* are ſuch as are either Expreſs'd by different Letters, or by the ſame Letters unequally repeated, as *a* and *b*: or *a* and *a a*: *b* and *b b b*, &c.

Like Signs are, when they are all of the ſame Nature, as all *Poſitive* or all *Negative*.

Unlike Signs are, when ſome of them are *Poſitive* or *Affirmatives*, and others *Negative*.

Simple Quantities are ſuch as conſiſt of but one Member; but *Compound ones* are ſuch as are Compounded of 2 or more Members Connected by the Signs + or —.

Coefficients are ſuch Numbers as are prefixed before any Letters: As in 5 *a*, 7 *c*. The *Coefficients* are 5 and 7: They are ſo called, becauſe they are ſuppoſed to help to make a Product or Rectangle, with the Quantity Expreſs'd by thoſe Letters: As will be farther Explained hereafter.

All Quantities expreſs'd by Letters which have no Number prefix'd before them, are ſuppoſed to have 1. for a *Coefficient*: Becauſe every thing contains it ſelf once: Thus, *b* is the ſame as 1 *b*.

If a Letter or Quantity have not the Negative Sign — before it, 'tis always ſuppoſed to have the Affirmative one +.

ADDITION.

ADDITION in *Algebra* or *Species*, is performed in general, by conjoyning the Quantities propos'd, preſerving their proper Signs. And the proper Mark or Sign of Addition is +: Which is always ſuppoſed to belong to the Quantity which follows it.

$\left.\begin{array}{l}\text{Thus if to } 3a\\ \text{you add } 2a\end{array}\right\}$ the Sum is $3a + 2a$ or $5a$.
and $A + 2b$ when added to $C + bb$ makes $A + C + 2b + bb$.

Addition in Algebra may eaſily be learnt by obſerving the following particular Rules.

RULE I.

When *Simple* and *Like* Integers having *Like* Signs are to be added, collect the Numbers (or Coefficients) all into one Sum, and to that Sum annex the Letters by which any of the Quantities was expreſs'd, and laſtly prefix the proper Sign.

$$\begin{array}{lr}\text{Thus} & -b\\ & -2b\\ \hline \text{make} & -3b\end{array}\qquad\begin{array}{lr}\text{and} & +bcd\\ & +2bcd\\ & +4bcd\\ \hline \text{make} & 7bcd\end{array}\qquad\begin{array}{lr}\text{and} & -36de\\ & -4de\\ \hline \text{make} & -40de\end{array}$$

RULE

Examples of Addition

$-c$	$-fghk$	$+12abcd$
$-3c$	$-fghk$	$+24abcd$
$-7c$	$-9fghk$	$+36abcd$
$-10c$	$-10fghk$	$+48abcd$
$-21c$.	$-21fghk$.	$+120abcd$.

$-3g$	$+lmn$	$-6pqrst$
$+3g$	$-lmn$	$+6pqrst$
0 0	0 0 0	0 0 0 0

RULE II.

When two Simple and like Quantities have equal Numbers prefixt, and unlike Signs, the Sum is 0.

$$\text{Thus,}\quad \begin{array}{r} +3a \\ -3a \\ \hline 00 \end{array} \qquad \text{and} \begin{array}{r} -bb \\ +bb \\ \hline 00 \end{array} \qquad \text{and} \begin{array}{r} -7dee \\ +7dee \\ \hline 00 \end{array}$$

N. B. The Reaſon of which is plain, if you conſider that all Quantities having negative Signs, are in Nature directly contrary to ſuch as have Affirmative ones: And therefore will always deſtroy one another. Thus, if a Man have 10 pounds in Caſh, and run in Debt 10 *l.* that is, if to his Caſh he add a — 10 *l.* (which is the proper way to expreſs a Debt) there will remain nothing: For the Debt or — 10 *l.* will deſtroy the Caſh or + 10 *l.* So alſo if a Man owe 10 *l.* and have nothing to pay it; then hath he a — 10 *l.* or is 10 *l.* worſe than nothing: And if any Perſon give him 10 *l.* or add a + 10 *l.* to his — 10 *l.* the Sum will be nothing; but however the Man will, tho' worth nothing, be 10 *l.* better than he was before.

So that 'tis a general Rule in Algebra, that to add —, is the ſame thing as to take away +, and to take away —, is the ſame thing as to add +, and to take away + is all one as to add —.

RULE III.

When two Simple and like Quantities are given, having unlike Signs, and unequal Numbers prefixt; ſubtract the leſſer Quantity from the greater; and to the Remainder annex the Letters due, prefixing the Sign that belongs to the greater Quantity.

Thus, $\begin{array}{r} +3a \\ -\ a \\ \hline +2a \end{array}$ and $\begin{array}{r} -8b \\ +2b \\ \hline -6b \end{array}$

The Reaſon of which is clear from what was ſaid in the *N. B.* of the laſt Rule.

RULE IV.

When three or more Simple and like Quantities have unlike Signs, Collect the Affirmative Quantities into one Sum, and the Negative into another, then proceed as in the third Rule; and the difference between them is the Sum ſought.

Thus, $$\left.\begin{array}{r} -7a \\ -3a \end{array}\right\} = -10a$$
$$\left.\begin{array}{r} +5a \\ +9a \end{array}\right\} = +14a$$
$$4a = \text{Sum.}$$

$$\begin{array}{ll} +abcd & -11nopqrst \\ -2abcd & +4nopqrst \\ \hline -abcd & -7nopqrst. \end{array}$$

$$\begin{array}{lll} +nrs & +lmno & -12dfgh \\ -2nrs & +3lmno & -4dfgh \\ +6nrs & -5lmno & +19dfgh \\ -9nrs & +8lmno & -3dfgh. \\ \hline -4nrs & +7lmno & 0\,0\,0\,0 \end{array}$$

$$\begin{array}{ll} +3b & +6h \\ +7d & -10n \\ +6g & +14m \\ \hline 3b+7d+6g & 6h-10n+14m \end{array}$$

$$\begin{array}{c} 7ee-6dd \\ 3k+4d \\ -8d+2k \\ \hline 7ee-6dd+3k+4d-8d+2k \\ \hline \end{array}$$

Contracted $7ee-6dd-4d+5k$

RULE V.

When two ot more Simple and unlike Quantities are propoſed, write them down one after another, without altering their Signs.

Thus,

$$\begin{array}{r} +3a \\ +4b \\ \hline +3a+4b \end{array}$$

From due Apprehenſion of, and mature Conſideration on which Rules, the Addition of Compound Quantities may be eaſily performed. Thus,

$$\begin{array}{rl} & +3ee+7bb \\ & -\;\;ee-2bb \\ & +\;\;ff+3ff \\ \hline \text{Sum,} & 3ee+7bb-ee-2bb+ff+3ff. \\ \hline \text{Contracted} & +2ee+5bb+3ff. \end{array}$$

Addition of Indexes, is performed after the ſame Manner as that of Algebraical Quantities.

Thus: To 3 add 3, the Sum is 6; where both are the Indexes of Integer Numbers: But to 3 add $\overline{2}$, the Sum will be 1; to $\overline{5}$ add 3, the Sum will be $\overline{2}$, &c.

SUBTRACTION.

SUBTRACTION in the General, is taking a leſſer Quantity out of a greater, in order to find the Difference between them, which Difference is in Common Arithmetick, uſually call'd the *Remainder*, as the leſſer Quantity to be Subtracted, is call'd the *Subtrahend*. The Difference is uſually Noted with x or d.

The general Mark or Sign of Subtraction in Algebra, is —, which whenever it comes between two Quantities, belongs to the latter of them.

Subtraction in Algebra is perform'd by conjoining the Magnitudes propoſed together, but always changing the Signs of the Subtrahend.

Thus: If from $4a$ you would Subtract a; by changing the Sign of the Subtrahend, it will ſtand thus:

$$4a - a = 3a; \text{ or thus, } \begin{array}{r} 4a \\ -a \\ \hline x = 3a \end{array}$$

So alſo, If from $6b - 54bb + 4fg$, you were to Subtract $6b - 54bb - 4fg$; there would remain only $8fg$; for from $6b - 54bb$

$-4fg$,

Examples of Subſtraction.

$$\begin{array}{r} 4abcd \\ -2abcd \\ \hline x = 2abcd \end{array}$$

From $3gfhkl$
take $2mnopq$

$x = 3gfhkl - 2mnopq$

From $21ee - bc + kl$
Take $20ee + bc - kl$

$$\begin{array}{r} 21ee - bc + kl \\ -20ee - bc + kl \\ \hline x = ee - 2bc + 2kl \end{array}$$

$-4fg$, taking the ſame Quantities by changing of their Signs, or adding them to it with their Signs already chang'd, and Comparing and Contracting them as taught in Addition, there will remain nothing but $8fg$.

For to Subtract $+$ is the ſame as to add $-$ and to Subtract $-$ is all one as to add $+$.

The Reaſon of which will be plain from the Inſtance given in the *N. B.* of Addition, and now applied to Subtraction. Suppoſe a Man have but 10 *l.* in Caſh: 'Tis plain, that if from him you take 10 *l.* he can have nothing left, which is Common Subtraction. Or if you make him run into Debt 10 *l.* that is, add to his real Caſh a — 10 *l.* he will ſtill be worth nothing in Reality. But if any one will pay that 10 *l.* for him, or which is all one, take away the Debt of 10 *l.* or Subtract the — 10 *l.* he doth as much ſervice as if he added a real 10 *l*, to his Caſh. Wherefore all manner of Subtraction in Compound Quantities may eaſily be perform'd by only obſerving the general Rule of Subtraction, to change all the Signs of the Subtrahend; and then comparing the ſeveral Members together and Contracting them.

One Inſtance is enough: Suppoſe from $36d + 5nn - 72bb$, you were to take $30d + 5nn - 72bb$, write them down one under another, changing all the Signs of the lower Rank. Thus,

$$\begin{array}{rrr} 36d & +5nn & -72bb \\ -30d & -5nn & +72bb \\ \hline 6d & & \end{array}$$

And then comparing them together, you will find all Deſtroy'd, or to Vaniſh by the contrariety of the Signs, but 6 *d.* which therefore is the true Difference between thoſe Quantities.

Subtraction of Indices, is done as in Algebraick Quantities, by changing the Signs of the Subtrahend. Thus, if from 3, the Index of the Logarithm of an Integer Number, you take $\bar{2}$, the Index of the Logarithm of a Fraction, the Difference will be 5; if from $\bar{3}$ you take $\bar{2}$, the Remainder will be $\bar{1}$; and from $\bar{3}$ taking 2, the Difference will be $\bar{5}$.

From $a - b + bb - c + dd - d + c$
Take $g + b - bb + cc - dd + d + c$

$$a - b + bb - c + dd - d + c$$
$$-g - b + bb - cc + dd - d - c$$

$$x = a - g + 2bb - 2b - cc + 2dd - c - 2d$$

From $hhdc - h + c - dd - cc + d$
Take $hdcc - h - c + dd - c + cd$

$$hhdc - h + c - dd - cc + d$$
$$-hdcc + h + c - dd + c - cd$$

$$x = hdch - hdcc \underset{\wedge}{\overset{+2c}{-}} 2dd - cc + d + c - cd$$

MULTIPLICATION.

FOR *Multiplication* in Algebra, the General Rule is, to conjoyn the Quantities propos'd by the Sign of Multiplication (×.) Which Sign, when the Quantities to be Multiplied are expreſs'd by but one or two Letters, is uſually omitted, and the Quantities written down like Letters in a Word, as you will find below.

The General Rule about the Sign is, that *like* Signs always give +, and *unlike* Signs, — in the Product. That is, if the Signs are either both Poſitive, or both Negative. the Product will always be Poſitive, but if one Factor be Poſitive and the other Negative, the Product is always Negative : The Reaſon of which follows below.

In Algebraick Multiplication, 'tis moſt commodious to begin to Multiply at the left Hand, becauſe we write that way.

Particular Rules.

I. If Two or more ſingle Quantities expreſs'd by Letters, whether *Like* or *Unlike*, are to be Multiplied into one another, and have no Numbers or Coefficients perfixt, joyn them together like Letters in a Word.

Thus, a Multiplied by b, makes ab; and

$$\begin{array}{llcll} & ab & \qquad & \text{and} & mno \\ \text{by} & dc & & \text{by} & qpr \\ \hline \text{Product} & abdc & & \text{Product} & mnopqr \end{array}$$

II. If two or more ſimple Quantities, whether Like or Unlike, are to be Multiplied, and have Numbers or Coefficients before them; firſt Multiply the Coefficients one into another, and then to the Product annex the Letters of both Quantities, ſo ſhall the new Quantity be the true Product.

Thus, If $3a$ were to be Multiplied by $4b$, the the Product will be $12ab$,

$$\begin{array}{llcll} & 36mn & \qquad & & 15abc \\ \text{by} & 4b & & \text{by} & 9def \\ \hline \text{Product} & 144mnb & & \text{Product} & 135adbecf \end{array}$$

III. The Multiplication of Compound Quantities depends entirely on the Preceding Rules; only you muſt be ſure to Multiply every Member of one Factor into every one of the other, and obſerve the Rule above given about the Signs. Thus,

$$\begin{array}{ll} & a + d - c \\ \text{by} & g - b + f \\ \hline \text{Prod.} & ga + gd - gc - ba - bd + bc + fa + fd - fc. \end{array}$$

IV. When

Examples of Multiplication.

$$\begin{array}{r} abcd \\ efgk \\ \hline \end{array}$$

Prod: $abcdefgk$.

$$\begin{array}{r} 7ad \\ 8ab \\ \hline \end{array}$$

Prod: $56aadb$

$$\begin{array}{r} -ab \\ +cd \\ \hline \end{array}$$

Prod: $-abcd$

$$\begin{array}{r} -gh \\ -kl \\ \hline \end{array}$$

Prod: $+ghkl$

$$\begin{array}{r} +mn \\ +op \\ \hline \end{array}$$

Prod: $+mnop$

IV. When one and the ſame Quantity is Multiplied by its ſelf, the Product is call'd the Square of that Quantity; and if that Square be Multiplied again by the Root, the Product is called the Cube, &c. And this way of producing the Powers of Numbers or Quantity, is call'd by Dr. *Pell*, and ſome others, *Involution*.

Thus, a × by $a = aa$ the Square, and aa × by $a = aaa$ the Cube or third Power, and $aaa \times a = aaaa$, or a^4 the Biquadrat, &c.
'Tis the ſame thing in Compounds.

$$\begin{array}{l} a + b \\ a + b \\ \hline \end{array}$$

Product $aa + 2ab + bb =$ the Square of $a + b$, and if that Square be Multiplied again by $a+b$, it produces $aaa + 3aab + 3bba + bbb$, which is the Cube of $a + b$.

N. B. That in Algebraick Multiplication, like Signs muſt give a Poſitive Product, and unlike Signs a Negative one, may be thus Demonſtrated.

I. Since Multiplication is only adding one Factor (or the Multiplicand) to its ſelf as often as there are Unites in the other, or in the Multiplicator: Therefore + Multiplying +, muſt produce +; ſince the Sum ariſing from the Addition of Poſitive Quantities, muſt be Poſitive.

II. A Quantity with an Affirmative Sign, Multiplying one that hath a Negatine one, muſt Produce a Negative Product; for 'tis only adding the Negative Factor to it ſelf as often as there are Units in the other. Now, never ſo many Negatives added together, will ſtill be Negative; and ſo the Product muſt have a Negative Sign.

$$\begin{array}{r} \text{Thus,} - 6 \\ \text{by} + 2 \\ \hline - 12 \end{array}$$

gives — 12 in the Product, becauſe 'tis only taking — 6, as often as there are Units in 2, *i. e.* twice; therefore the Prod. muſt be — 12.

III. Negative Quantities Multiplying Poſitive ones, muſt produce a Negative Product; becauſe, in this Caſe the *Multiplicator* havivg a Negative Sign, works on the *Multiplicand* by Subtraction; which therefore muſt be Subtracted or made Negative (by changing its Sign) as often as there are Negative Units in the Multiplier. Thus, If + 6 be Multiplied by — 2, the Product here alſo muſt be — 12; becauſe the Multiplicatnr 2 having a Negative Sign, ſhews that the Multiplicand muſt be Subtracted twice from reality, or twice repeated with a Negative Sign; wherefore the Product muſt be Negetive.

$$\begin{array}{r} + 6 \\ - 2 \\ \hline \text{Product} - 12 \end{array}$$

ab − c multiply'd by
c − g + h

Prod: cab − cc − gab + gc + hab − hc

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dd + c − d multiply'd by
$dd-c+d=ddd+dc-d\underset{\wedge}{d}^{+ddd+dc-dd}-cdd-cc+cd+ddd+dc-dd$

---

aa + b multiply'd by bb + a
$=\underset{\wedge}{b}^{b}aa+b\underset{\wedge}{b}^{b}+$ ~~[illegible]~~ $+aaa+ab.$ ~~[illegible]~~
~~move contracted 2baa+abb+aaa+ab~~

−gh − d + h multiply'd by
$d-gh=-dgh-dd+dh+ggh^{hh}+\underset{\wedge}{g}^{h}d-g\underset{\wedge}{h}^{h}+$

$$\begin{array}{r} -17 \\ -43 \\ \hline 51 \\ 68 \\ \hline +731 \end{array} \qquad \begin{array}{r} +19 \\ +13 \\ \hline 57 \\ 19 \\ \hline +247 \end{array}$$

$$\begin{array}{r} -12 \\ +2 \\ \hline -24 \end{array}$$

d
M
a
p
P
A
L
k
A

d
h
p
S
t
U
a

IV. Negatives Multiplying Negatives muſt produce an Affirmative or Poſitive Product; becauſe Multiplication by a Negative Quantity being only a Subtraction or changing the Sign of the Multiplicand as often as there are Units in the Multiplicator; and ſince Subtracting — is the ſame as Adding + (as was ſhewed in Subtraction) the Defect of the Multiplicand is by this means taken away; and conſequently the Product will be Affirmative.

Thus, If — 6 be Multiplied by — 2, the Product will be + 12; becauſe the Multiplicator 2 having here a Negative Sign, acts on the Multiplicand by Subtraction, Subtracting its defective Sign, or changing it into an Affirmative one, (for to Subtract — is to add +) as often as there are Units in it ſelf. Wherefore the Product muſt have a Poſitive Sign. *Q. E. D.*

DIVISION.

DIVISION in Algebra in the General, is Reducing the Dividend and Diviſor to the Form of a Fraction, which Fraction is the Quotient. Thus, if $a\,b$ were to be Divided by $c\,d$: It is in Algebra placed thus $\frac{a\,b}{c\,d}$ and this Fraction is the Quotient.

Some expreſs Diviſion thus $c\,d)\;a\,b$ or $a\,b \div c\,d$, which Character $\div$ is the ordinary Sign of Diviſion.

For duly performing the Work of Algebraick Diviſion, obſerve theſe Rules:

I. When the Dividend is equal to, or the ſame with the Diviſor, the Quotient is 1. (not 0.) For every thing is it ſelf once. Therefore when ever this happens, as it will often do in Equations; remember always to place 1. in the Quotient.

II. When the Quotient is expreſs'd Fraction-wiſe, (as in Simple Diviſion) if the ſame Letters are found equally repeated both above and below the Line of Separation, you may caſt off thoſe equal Letters, and the Remainder will be the true Quotient. Thus,

Examples of Division

$$\frac{abcd}{fcd} = \frac{ab}{f}$$

$$\frac{afnh}{afnh} = 1$$

$$\frac{36dclp}{6lp} = 6dc$$

$$\frac{-49dde}{-7de} = 7d$$

$$\frac{-51rstu}{3rs} = -17tu$$

$$\frac{72ghklm}{-12hklm} = -\frac{6}{1}g.$$

$$\frac{ab}{db} = \frac{a}{d} \qquad \text{and } \frac{abc}{ab} = c, \text{\&c.}$$

The Reaſon of which is plain; becauſe the Fraction $\frac{a}{d}$ being Multiplied both above and below the Line by b (in the firſt Inſtance) hath not its Value at all altered thereby, and therefore when the Quantity $\frac{ab}{db}$ comes to be Divided by the common Multiplier b, which is done by caſting off b, the ſame Value will remain and $\frac{a}{d}$ is the Quotient.

III. When the Quantities expreſs'd by Letters have any Coefficients, Divide them as in Common Arithmetick, and to the Quotients annex the Quantities expreſs'd by the Letters. Thus,

$$\frac{360\, ab}{24\, b} = 15\, a$$

Only remember that if the Quantities have unlike Signs, the Quotient muſt have a Negative Sign, if like Signs a Poſitive one.

IV. The General way of Diviſion of Compound Quantities is like the ordinary way in Common Arithmetick: Reſpect being had to the Rules of

Algebraic *Addition*, *Subtraction* and *Multiplication*: Obſerving always this, That like Signs give + and unlike — in the Quotient, as was ſaid before.

You muſt always take Care to Divide every Member or part of the Dividend, by its *proper Diviſor*: (*i. e.* by ſuch an one whoſe Letters ſhew it to be of the ſame Kind with the other): For you muſt always place ſuch a Letter in the Quotient, as will, when Multiplied into the Diviſor, produce the Dividend (or at leaſt a good part of it) ſince the Dividend is a Rectangle under the Diviſor and Quotient. Thus,

$$\begin{array}{lll}
a+b) & aa+ab-ca-cb & (a-c \\
 & \underline{aa+ab} & \\
 & 0 \quad -ca-cb & \\
 & \quad\;\; \underline{-ca-cb} & \\
 & \qquad\quad 0 &
\end{array}$$

Another Example.

$$\begin{array}{lll}
zz-16) & z^6-8z^4-124zz-64 & (z^4+8zz+4 \\
 & \underline{z^6-16z^4} & \\
 & \quad 8z^4-124zz & \\
 & \quad \underline{8z^4-128zz} & \\
 & \qquad\quad 4zz-64 & \\
 & \qquad\quad \underline{4zz-64} & \\
 & \qquad\qquad 0 &
\end{array}$$

Divide

$dd+dc+ggd+ggc+2c+2d$ by $d+c$

$d+c)\ dd+dc+ggd+ggc+2\overset{+2d}{c}\ (d+gg+2$

$dd+dc$

0

$ggd+ggc$

$ggd+ggc$

0

$2c+2d$

$2c+2d$

0

Divide

$bbaa+bbb+aaa+ab$ by $bb+a$

$bb+a)\ bbaa+\overset{aaa}{\cancel{bbb}}+\overset{bbb}{\cancel{aaa}}+ab\ (aa+b$

$bbaa+aaa$

$0+bbb+ab$

$bbb+ab$

Divide

$aa+ab-ca-cb$ (by $a-c$

$a-c)\ aa+ab-ca-cb\ (a+b$

$aa-ac$

$ab+ac-ac-bc$

$ab-bc$

0

Another Example.

$$3dd - ee)\,9d^4 + 12\,d^3e - 4dee - eeee\,(3dd + 4de + ee)$$

$$9d^4 - 3ddee$$

$$12\,d^3e + 3ddee - 4dee$$

$$12d^3e \qquad - 4dee$$

$$3ddee - e^4$$

$$3ddee - e^4$$

$$0$$

N. B. That the ſame Reaſon for like Signs giving a Poſitive Quotient, and unlike a Negative one, holds here as well as for the Product in Multiplication, is clear, from conſidering the Nature of Diviſion.

For every Dividend being nothing but a Product made by Multiplying the Quotient by the Diviſor ; the Sign of each Factor muſt be ſuch, as according to the former Rules in Multiplication, can produce the Dividend. Wherefore if the Dividend be Poſitive, and Divided by a Negative, the Quotient muſt be Negative ; Since if it be Poſitive, it cannot produce the Dividend by Multiplication into the Diviſor. If the Dividend be Negative, either the Diviſor or Quotient muſt have a Negative Sign ; but they cannot be both Negative. For then they would produce a Poſitive Dividend.

Wherefore 'tis plain that, if the Dividend be Divided by a Quantity, which hath a *like* Sign with it, the Quotient muſt be *Poſitive* ; but if by one having an *unlike* Sign, the Quotient will be *Negative.*

OF
FRACTIONS.

A *Fraction* is a broken Number, or Quantity, expreſſing the Parts of ſome Integer. It conſiſts of two Parts with a Line of Separation placed between them, Of which, that above the Line is call'd the *Numerator*, becauſe it *Enumerates* or tells you how many of the Parts of the Integer, the Fraction contains: And that below the Line is call'd the *Denominator*; becauſe it *Denominates* or Expreſſes the Nature of the Parts the Integer is ſuppoſed to be divided into. Thus,

Suppoſe $a = 3$ and $b = 4$, then will $\frac{a}{b}$ or $\frac{3}{4}$ be a Fraction, expreſſing, that ſome Integer being divided into 4 Parts or *Quarters*, there is taken 3 of them, or 3 Quarters.

A Fraction is either *Proper*; when the *Numerator* is leſs than the *Denominator*; as $\frac{3}{4}$: Or *Improper*, when the *Numerator* is equal to it, or greater: As $\frac{4}{4}$ or $\frac{4}{3}$ are Improper Fractions; be-

cauſe

Examples of y^e

Extraction of y^d square & cube roots.

Extract y^e square root of

$(28+4\partial+15+9+\frac{12}{4}\ \frac{1}{2}$

$2\partial)\ 4\partial\partial - 8\partial + 64 + 24\partial + 14\partial - (2\partial - 2 + 6 +$

$(+14$

$4\partial\partial$

$4\partial - 2)\ 0 - 8\partial + 64$

$-8\partial + 4$

$4\partial - \overset{+6}{4})\ 60 + 24\partial$

$24\partial - 24 + 36$

$4\partial -$ [illegible]$)$ [illegible] $+ 14\partial + 48$

$14\partial + 28 + 12\frac{1}{4}$ $\quad +24\frac{1}{4}$

$4\partial - 12 + 7)\ 72$ [illegible] $- 28 + 4\partial + 15 + 9$

$4\partial + 15 +$ [illegible]

$0\ 8 + 21\frac{1}{4}$

Extract the square root of

$ab)\ aabb - 2aab + aa + 4ab - 4a + 4\ (ab - a +$

$aabb$

$2ab - a)\ 0 - 2aab + aa$

$-2aab + aa$

$2ab - 2a + 2)\ 0 + 4ab - 4a + 4$

$4ab - 4a + 4$

0

cauſe one Expreſſes the whole *Integer*, and the other more than the *Integer*; however 'tis often of good uſe to expreſs Quantities after this way.

The Operations about Algebraick Fractions, or Fractions expreſs'd by Letters, are much of the ſame Nature with thoſe in Common Arithmetick.

I. All Fractions ought firſt to be reduced to their loweſt Terms; which is done by dividing both Numerator and Denominator, by their *greateſt Common Diviſor*; that is, the *greateſt* Quantity which can divide both. For then the Quotient will be a Fraction of the ſame value as the former, but in the ſmalleſt Terms that can be. Thus, $\frac{3aa}{6a}$ by dividing bothParts by $3a$, will be brought down to $\frac{a}{2}$ or $\frac{1}{2}a$ and $\frac{4a^6}{6a^4}$ being divided by its greateſt Common Diviſor $2a^4$, will be reduced to. $\frac{2aa}{3}$

$$\begin{array}{l} 4z) \\ 4z) \end{array} \quad \frac{36zz}{4bz+16dz} \quad \left(\frac{9z}{b+4d} \right.$$

And this may moſt times be done by Inſpection, by caſting out of both *Numerator* and *Denominator*, ſuch Letters as are Multiplied into both of them, as in theſe Examples.

But ſuch greateſt Common Diviſor may be found in all Caſes, where the Eye cannot readily diſcover it, by dividing the Denominator by the Numerator, and the laſt Diviſor by the Remainder, if any be; and ſo on, until there come to remain nothing: And then that laſt Diviſor is the greateſt Common Meaſure. But if Unity, or 1 remain at laſt, then the Fraction was in its loweſt Terms at firſt, and cannot be reduced to any ſmaller Terms. This Practice is the ſame as in Vulgar Fractions; and you have an Example of it in *Species* in *Ward*'s Algebra, *Chap.* 4.

II. To reduce any Integer, as b or $a+c$ to the Form of an Improper Fraction, draw the Line of Separation, and under it write 1. then it will ſtand $\frac{b}{1}$ or $\frac{a+c}{1}$, which tho' in the Form of Fractions, are not altered, becauſe 1 neither Multiplies nor Divides.

If a *Denominator*, as d were given: Firſt, Multiply the given Integer by ſuch Denominator, and then write the Denominator under the Product. Thus,

$$\frac{db}{d}=b \quad \text{and} \quad \frac{da+dc}{d}=a+c$$

III. To reduce Fractions of different Denominators, to others of the ſame Value, that ſhall have a Common Denominator; (*which Operation muſt always precede Addition and Subtraction in Fractions.*) You muſt firſt bring the Fractions down as low as you can; (by Rule 1.) then Multiply a-croſs the

Extract the Cube root of

$$aaa+3aab+3aa+3abb+6ba+3a+bbb+3bb\overset{+1}{+3b}$$

$$aaa+3aab+3abb+bbb+3aa+6ba+3bb+3a\underset{\wedge}{+}\overset{3}{}b \qquad (a+b+1$$

$$\underline{aaa}$$

$$\underline{0+3aab+3abb+bbb}$$

$$\underline{3aa+3a}$$

$$\underline{3aab+3abb+bbb}$$

$$\underline{0\ 3aa+6ba+3bb+3a+3b+1}$$

$$\underline{3aa+6ab+3bb}$$

$$\underline{3aa+6ba\underset{\wedge}{+}\overset{3}{}bb+3a+3b+1}$$

$$0$$

Extract y^e cube root of

$$aaa-3aab+3abb-bbb+3aa-6ab+3bb+3a\underset{\wedge}{}\overset{-3b+1}{} \ (a-b+1$$

$$\underline{aaa}$$

$$\underline{0-3aab+3abb-bbb}$$

$$\underline{3aa+3a}$$

$$\underline{-3aab+3abb-bbb}$$

$$\underline{0+3aa-6ab+3bb+3a-3b+1}$$

$$\underline{3aa-6ab+3bb+3a-3b}$$

$$\underline{3aa-6ab+3bb+3a-3b+1}$$

$$0$$

the Numerator of the firſt into the Denominator of the ſecond for a new Numerator for the firſt Fraction, then the Numerator of the ſecond into the Denominator of the firſt, for a new Numerator for the ſecond Fraction; and laſtly, Multiply the Denominators one into another, for a Common Denominator. Thus, let $\frac{a+b}{d}$ and $\frac{hh}{f}$ be given: and they will by this Rule be reduced to $\frac{fa+fb}{df}$, and $\frac{dhh}{df}$: Fractions in Value equal to the former.

The Reaſon of which is plain; for each Fraction is Multiplied and Divided by the ſame Quantity or Letter, and therefore muſt retain the ſame Value as before, tho' reduced to another Form:

$$\frac{4}{6} \qquad \frac{3}{4}$$

$$\frac{16}{24} \qquad \frac{18}{24}$$

For every Fraction being Multiplied by Multiplying its Numerator, but Divided by Dividing it; and being alſo Multiplied by Dividing the Denominator, and Divided by Multiplying it.

It follows, That each Fraction will gain as much by the Multiplication of its Numerator, as it loſes by the Multiplication of its Denominator: And *Vice verſa*, in caſe of Diviſion by one and the ſame Quantity.

If there are more than two Fractions, every Numerator muſt be Multiplied continually into all the Denominators, but its own; and the Denominators one into another continually for a new Denominator. *E. gr.* $\frac{a}{x}$, $\frac{b}{y}$, $\frac{c}{z}$, will be reduced to this Form $\frac{ayz}{xyz}$, $\frac{bzx}{yzx}$, $\frac{cyx}{zyx}$, which are Fractions of the ſame Value as the former (as is apparent by ejecting the Common Letters) but reduced to a Common Denominator.

IV. And when this is once underſtood, *Addition* and *Subtraction* in *Fractions* are performed by only Adding or Subtracting the *Numerators*, and Subſcribing the *Common Denominator* before found, Thus,

If the Fractions $\frac{a+b}{d}$ $\frac{bb}{f}$ were to be *Added* or *Subtracted*; they will ſtand, when reduced, (by *Rule* 3.) in this form, $\frac{fa+fb+fbb}{df}$, or $\frac{fa+fb-fbb}{df}$: The former of which, is the *Sum*, the latter the *Difference* of the two given *Fractions*.

V. *Multiplication in Fractions*, is perform'd by Multiplying the Numerators into one other, for a new Numerator, and the Denominators for a new Deno-

Fractions.

1 What is y^e greatest common divisor of $9ank + 168an$ & $3an$?

$3an)\ 9ank + 168an\ (3k + 56$
$9ank$
$0 + 168an$
$168an$
0

What is y^e greatest common divisor of $2bd+n$ & $bd+n$, supposing $bd = 2n$.

$bd+n)\ 2bd+n\ (1$
$bd+n$
$bd)\ bd+n\ (1$

What is $\frac{bn}{n\ (\ \frac{bd}{n}}$

the greatest common divisor of $3aum+16+cd$ & $aum+4+d$ Supposing $aum+4+d = 2aum+12+cd-d$

$aum+4+d)\ 3aum+16+cd\ (1$
$aum+4+d$
$2aum+12+cd-d)\ aum+4+d\ ($
~~$2aum+12+cd-d$~~
~~$(aum+4+d$~~

2 Reduce $adnk-3$ to y^e form of an improper fraction. $\frac{adnk-3}{1}$

3 Reduce $\frac{auy-e}{o}$ & $\frac{df}{m}$ to fractions of a common denominator.

$\frac{auy-e}{c}$ $\frac{df}{m}$

$\frac{auym-em}{om}$ $\frac{dfo}{om}$

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Denominator, the Fractions having been firſt reduced to their loweſt Terms. Thus,

$$\frac{a}{b}\times\frac{d}{c}=\frac{da}{bc} \text{ and } \frac{a+b}{c}\times\frac{a-b}{d}=\frac{aa-bb}{cd}$$

Hence, If any Fraction be Multiplied by the Denominator, or by ſome Integer, the ſame with it, the Numerator is the Product. As $\frac{aa}{b}\times b=aa$, for $\frac{aa}{b}\times\frac{b}{1}=\frac{aab}{b}$; which, caſting off the Common Letters in both Parts, leaves aa.

Alſo if any Fraction be to be Multiplied by ſome Letter or Letters that are found in every Member of the Denominator; the Multiplication may be made only by ejecting ſuch Letters out of the Denominator: As $\frac{ab}{cd}$ Multiplied by $d=$ to $\frac{ab}{c}$.

VI. *Diviſion in Fractions*, is perform'd (after Reduction according to *Rule* 3.) by Multiplying the Numerator of the Dividend, by the Denominator of the Diviſor, for a *Numerator*; and the Denominator of the Dividend by the Numerator of the Diviſor, for a new *Denominator*. As in *Vulgar Fractions*. Thus,

$$\frac{a}{b}\Big)\,\frac{d}{c}\,\Big(\frac{bd}{ac}$$

The Reaſon of which is plain, from what was ſaid above, That a Fraction is divided by Multiplying its Denominator. Thus,

$$\frac{3}{4}\Big)\ \frac{12}{16}\ \Big(\frac{48}{48}=\Big)$$

For to Divide $\frac{12}{16}$ by $\frac{3}{4}$, is to ſeek how often 3 the Numerator of the Diviſor, is in $\frac{12}{16}$; which is done by Multiplying 16 by 3, and the Anſwer is $\frac{12}{48}$: But then again, becauſe $\frac{3}{4}$ is but $\frac{1}{4}$ of 3, it will be contained in $\frac{12}{16}$, 4 times oftner than 3 is; and therefore in order to bring it to a *Par*, Divide the Value of that Fraction by Multiplying its Denominator by 12, and the Product 48 will be the Numerator of the Quotient.

But if it happen that the Fractions have a Common Denominator, then caſt off that, and divide one Numerator by the other. Thus,

$$\frac{a}{b}\Big)\ \frac{c}{b}\ \Big(=\frac{c}{a}\qquad\text{and}\ \frac{b}{a}\Big)\ \frac{bb}{a}\ \Big(=b$$

For Fractions having a Common Denominator are as their Numerators.

VII. A Mixt Quantity or Number, is that which is part Integer, and part Fraction. As $aa+\frac{b}{c}$: Such Quantities are reduced to the form of improper Fractions, by firſt Multiplying the Integral Part by the Denominator of the Fractional Part, then adding the Numerator to it, and

4 Add $\frac{rstu}{\partial}$ & $\frac{abc}{h}$ together.

$$\frac{rstu}{\partial} \quad \frac{abc}{h}$$

$$\frac{abc\partial}{\partial h} + \frac{rstuh}{\partial h} = \frac{abc\partial + rstuh}{\partial h}$$

5 Substract $\frac{aa+b}{l}$ from $\frac{bb+a}{p}$

$$\frac{aap+pb}{lp} - \frac{bbl+al}{lp} = \frac{aap+pb-bbl+al}{lp}$$

6 Multiply $\frac{tnb}{y}$ by $\frac{lal}{mn}$

$$\frac{tnb}{y} \times \frac{lal}{mn} = \frac{tnblal}{ymn}$$

7 Divide $\frac{b+a+u}{c}$ by $\frac{hk}{e}$.

$$\frac{hk}{e}\Big)\,\frac{b+a+u}{c} = \frac{be+ae+ue}{chk}$$

$$\frac{hke}{ce}\Big)\,\frac{be+ae+ue}{ce} = \frac{be+ae+ue}{hke}.$$

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and Subſcribing the Denominator under all. Thus the former Quantity $aa + \frac{b}{c}$ is Reduced to this improper Fraction $\frac{caa+b}{c}$.

Every Improper Fraction is Reduced back again into its equivalent Mixt Number, or Integer, by dividing the Numerator by the Denominator. Thus, $\frac{caa+b}{c}$ Divided by c; quotes $aa + \frac{b}{c}$; and $\frac{aa}{1}$ divided by 1, makes aa.

OF
EQUATIONS.

AN *Equation* in *Algebra*, is the mutual comparing of two equal Things of different Names or Denominations: As, ſuppoſe 3 Pounds equal to 60 Shillings = 720 Pence, which is equal to 2880 Farthings, *&c.* it may be written thus, $3\,l. = 60\,s. = 720\,d. = 2880\,f.$

The *Terms of an Equation*, are the ſeveral Quantities or Parts of which every Equation is compoſed, connected together by the Signs + and —. As in this Equation $a = b + c$. The Terms are a, b and c; where 'tis ſuppoſed that ſome Quantity repreſented by a, is equal to the Sum of b and c; or to b and c added together.

Whenever a Queſtion or Problem is propoſed in *Algebra*, we always ſuppoſe the thing *ſought* or *required*, to be *known* or *done*.

And then by putting the Letter a, or ſome other *Vowel* (many uſe the laſt Letters of the Alphabet, x, y, z) for the unknown Quantity, or for the thing ſought; and *Conſonants* for whatever is *known* or *given*, in order to diſtinguiſh one from the other: The Queſtion or Problem is firſt *throughly conſider'd*, and then duly *ſtated*, and after this Judiciouſly

II In this Question (what numbers are these, ye former of which being doubled & added to ye latter & that added to 5 times 10 & ye sum of all divided by ye former number added to the latter added to 10 = $3\frac{1}{2}$) there are 3 equations, viz. (1) $b=10$ (2) $\frac{5b+2a+u}{e}=3\frac{1}{2}$ (3) $b+a+u=e$, & 3 unknown numbers, viz. a, u, & e, so that there will be several answers to the Question, for any number between 16 & 20 $=e$, as may be found thus,

	1	$b=10$
	2	$\frac{5b+2a+u}{e}=3\frac{1}{2}$
	3	$b+a+u=e$
$2\times e$	4	$5b+2a+u=3\frac{1}{2}e$
$4\times\frac{2}{1}$	5	$10b+4a+2u=7e$
5,3,	6	$10b+4a+2u=7b+7a+7u$
$6-7b$	7	$3b+4a+2u=7a+7u$
$7-4a+2u$	8	$3b=3a+5u$
$8\div\frac{3}{1}$	9	$b=a+u+\frac{2}{3}u$
$9\div\frac{5}{3}$	10	$\frac{3b}{5}=\frac{3a}{5}+u$
3,10.	11	$e=b+\frac{3b}{5}+\frac{2a}{5}$
3,9.	12	$e=2b-\frac{2u}{3}$ so that e is more than 16 & less than 20.

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ouſly compared, transformed and varied by Addition, Subtraction, Multiplication, Diviſion, Extraction of Roots, *&c.* according as the Nature of the Thing, and the Rules of Art direct; till at laſt the Quantity *ſought*, or at leaſt ſome Power of it, becomes equal to ſome *known* or *given* Quantity, and ſo is it ſelf of conſequence diſcovered.

After a Queſtion is duly Stated, 'tis proper to conſider whether it be ſubject to any *Limitations*, or not. To which End the Writers of *Algebra* give theſe General Rules.

I. If the Quantities ſought or required, are more than the Number of the given Equations, the Queſtion is capable of Innumerable Anſwers. See *Kerſey*'s *Algebra*, P. 301. Vol. 1.

II. But if the given Equations, independant one upon another, are juſt as many, as the Quantities ſought: Then the Queſtion hath only one certain and determinate Number of Anſwers.

If the Quantities ſought or required, are leſs in Number than the given Equations, the Queſtion is yet more Limited; and ſometimes it is diſcoverable, that 'tis not to be reſolved, by reaſon of ſuch Equations, being inconſiſtent with each other.

Equations, in order to be Reſolved, muſt firſt be *Prepared* and *Reduced*; which is uſually done by the following Rules, or ſuch like.

I. If the Quantity ſought, or any *Part* or *Degree* of it be in Fractions, let all be reduced to one Common Denomination; and then omitting the Denominators, let the Equation be continued in the Numerators only; or in Practice, Multiply the whole by the Denominator of the Fractional part.

V. gr. $\frac{a+b}{c}+d=100=B$.

Then firſt, $\frac{a+b+cd}{c}=B$.

And then, $a+b+cd=cB$.

Or if, $a-b=\frac{aa+cc}{d}+b+b$.

Multiply all by d, and it will ſtand thus,

$$ad-db=aa+cc+db+db.$$

Or if, $a-75=\frac{3}{4}bb+c-g$.

Multiply all by 4, and

$$4a-300=3bb+4c-4g.$$

And this is call'd by *Vieta*, *Iſomeria*, and by others *Converſion*.

II. When there is an Intermixture of Quantities, known and unknown in any Equation: Let all the unknown Quantities (by *Tranſpoſition*) be

made

made to poſſeſs only one Side of the Equation, and all the known ones another. *Tranſpoſition* is always done by putting the Quantity over to the other Side with a contrary Sign to what it had before. The Demonſtration of which *Rule* depends on that *Axiom*, That if to or from Equals, you add or ſubtract Equals, the Sums or Remainders will be equal.

Thus ſuppoſe, $a - 34 = 60$.

Then, $a = 34 + 60 = 94$.

Or if $a + b - d = b + c + e$.

Then, $a - e = c + d$.

If, $4a - 300 = 3bb + 4c - 4g$.

Then, $4a = 300 + 3bb + 4c - 4g$.

If, $36 + 44 = a - 60$.

Then, $a = 140$.

III. If the higheſt Power or Species of the unknown Quantity, be Multiplied into any known Quantity or Quantities, let the whole be Divided by ſuch known Quantity or Quantities.

Thus if, $5aa = 30000$. $aa = 6000$.

If, $ba + ad = 100$.

Then, $a = \dfrac{100}{b + d}$.

If, $d\,e\,e + d\,d\,e = z$.

Then, $e\,e + d\,e = \frac{z}{d}$.

And this Operation is called by *Vieta*, *Paraboliſmus*, by others *Depreſſion.*

IV. If all the known Quantities happen to be Multiplied into any degree of the Unknown one; let all be brought down (by *Diviſion*) to the loweſt degree thereof that can be.

As if, $a\,a\,a\,a + b\,a\,a\,a = z\,z\,a\,a$.

Then by Diviſion of all, by $a\,a$.

$a\,a + b\,a = z\,z$.

If $a\,a + a\,b - a\,c = a\,d - f\,a$.

Then will $a + b - c = d - f$. By Diviſion,

And, $a = d - b + c - f$, by Tranſpoſition.

If $e\,e + 9\,e - 7\,e = 15\,e + 34\,e - 10\,e$.

Then will $e + 9 - 7 = 15 + 34 - 10$. by Diviſion; and $e + 2 = 39$.

And farther by *Tranſpoſition*, $e = 37$.

And this is that *Rule* which *Vieta* calls *Hypobibaſmus.*

If any one Member of the Equation be [illegible], all must be raised up to that Power, and then the Equation [illegible].

EXAMPLE.

Then by Transposition,

[illegible]

Then by the Rule,

[illegible]

And by [illegible] the other,

[illegible]

NOTA.

[illegible]

RULE VI.

[illegible]

If, $y a a + y a d = q$.

Then, $a a + a d = \frac{q}{y}$.

And this Operation is called by some, Divi-ſion, by others Depreſsion.

IV. If all the known Quantities happen to be Multiplied into any degree of the Unknown, they be all be brought down (by Diviſion) to the low-eſt degree thereof that can be.

As if, $a a a a + b a a a = c q a a$.

Then by Diviſion of all, by $a a$.

$a a + b a = c q$.

If $a a + a b - a c = a d - f a$.

Then will $a + b - c = d - f$. By Diviſion.

And, $a = d - b + c - f$. by Tranſpoſition.

If $a + 9 a - 7 a = 12 a + 34 a$ [illegible]

Then will $1 + 9 - 7 = 12 + 34 -$ [illegible] Diviſion, and [illegible] $= 39$.

And farther by Tranſpoſition, $a = 37$.

And this is that Rule which ſome call [illegible] [illegible]

V. If any one Member of the *Equation* be a *Surd Root*, all muſt be raiſed up to that Power, and then the *Equation continued.*

If $\sqrt{b\overline{a}} + b = c.$

Then by *Tranſpoſition*,

$$\sqrt{ab} = c - b.$$

And by this Rule,

$$cc - 2bc + bb = ab.$$

And by Rule the third,

$$a = \frac{cc - 2bc + bb}{b}.$$

NOTE.

To Raiſe up any Quantity to the Power of another, is to Multiply it into it ſelf, or to Involve *it according as the Index of that Power directs. Thus becauſe* $\sqrt{}$: a b *ſignifies the Square Root of* a b, *therefore* a b, *without the Radical Sign will be a Square; and conſequently to continue the Equality,* c — b *muſt be Squar'd too.*

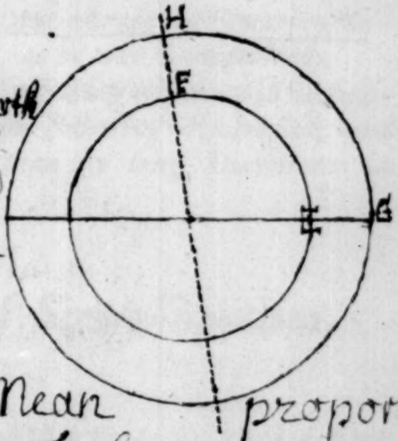
RULE VI.

Since whenever 4 Quantities are *diſcretely*, or 3 *continually Proportional*, the Rectangle or Product of the two mean Terms (or the Square of the middle one, when there are but 3) is equal to the Rectangle or Product of the Extreams (by the 12th of the 6th of *Euclid*'s *Elements*.) Therefore 'tis very eaſie, and often very uſeful to reſolve *Equations* into *Analogies* or *Proportionals*, and *vice verſâ*; which when well underſtood, opens the way for the *Geometrical Conſtruction*, and conſequently for one very good way of Reſolution of *Equations*.

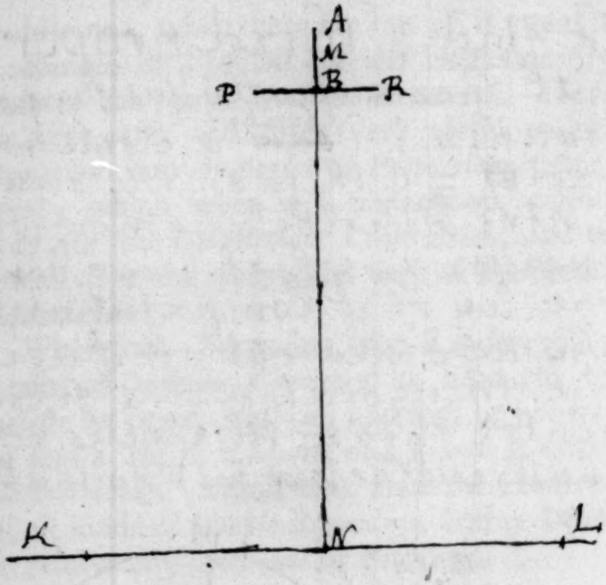
Wherefore, Suppoſing the Reader tollerably verſed in *Common Geometry*, as he ought to be, before he begin *Algebra*; and that he knows how to find a *Third*, a *Mean*, or a *Fourth Proportional*, *Geometrically*. I ſhall next ſhew the Conſtruction of all kinds of *Simple Equations*, before I proceed to reſolve any *Queſtions* or *Problems*.

To find a third proportional Geometrically, (as ſuppoſe $b:a::a:z$.) ~~draw~~ deſcribe a circle whoſe radius $= b$; then on the same centre deſcribe another whoſe radius $= a$; which being done, set one leg of ye compaſſes at E, & with it the other cut off ye arch EF $= a$. then draw a line from ye centre thro' both circles which ſhall cut ye leſſer in F, ſo ſhall GH $= z$.

To find a fourth proportional (as ſuppoſe $b:a::d:z$) do as before only let EF $= d$.



To find a Mean proportional (as ſuppoſe $b:a::a:z$, b & z being known,) draw a line in which make the diſtance AB $= \frac{b}{2}$ then ſet PR $= b$ at right angles to ye line, ſo as it may paſs thro B; then divide AB in ye midſt from which ſet MN $= z$, & thro' N draw ye line KL at right angles, then ſet the compaſſes fro A to N, & placing one leg in B with the other cut the line KL in K & L. then will KN or LN $= a$.



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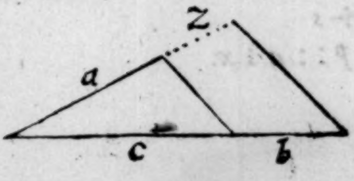
CONSTRUCTION OF *EQUATIONS*,

IN Algebra, *is the Contriving ſuch Lines and Figures as ſhall Demonſtrate the Equation, Canon, or Theorem, to be true Geometrically.*

The Method of effecting which, will be ſufficiently plain from the following Examples, and thence eaſier learn'd than by long Directions in Words.

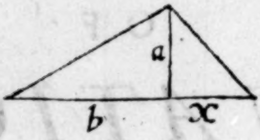
Conſtruction of Simple Equations.

I. IF $\frac{ab}{c} = z$, then $c.b :: a.z$. 12. *l.* 6 *Eucl.*



Or if, $\frac{aa}{b} = x$. Then, $b : a :: a : x$.

By 8 *e.* 6. *Euclid.*



Or if, $\frac{ab+ag}{b+1} = x$. Then, $b+1 : b+g :: a : x$.

Or if it were, $\frac{ab+ag}{b-1} = x$. Then, $b-1 : b+g :: a : x$

II. If $\frac{ab+mn}{r+s} = x$. The Conſtruction and Solution will be more difficult; becauſe no Letter in the Numerator is taken twice: But that it may be ſo, and that (a) for Inſtance, may be twice uſed; make as $a : n :: m :$ a fourth Proportional, which let be p. Then, $nm = ap$, and conſequently $\frac{ab+ap}{r+s} = x$. Wherefore as by *Rule* 1. $r+s : b+p :: a : x$.

If the Equation were propos'd $\frac{ab+mn}{p-q} = x$.

You must find out a middle Proportional between a and b, which suppose to be r. Also another middle Proportional between m and n, which let be s. Then will the Equation stand thus, $\frac{rr+ss}{p-q} = x$.

Let therefore a Right-angled Triangle be made, wherein the Perpendicular AP = r, and the Base AB = s. Then the side PB = $\sqrt{rr+ss}$, which side according to the Equation is to be divided by p − q. Make, as p − q to PB (= $\sqrt{rr+ss}$) so PB to a third Proportional, which shall be x sought.

Or if, $\frac{ymn}{b} = x$. Then, $b : n :: m :$ [illegible]

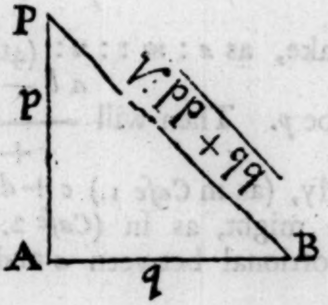
By [illegible]

Or if, $\frac{ab+ag}{2b+c} = x$. Then, $b+c : b+g ::$ [illegible]

Or it may be, $\frac{ab+ag}{b-c} = x$. Then, $b-c : b+g :: a : x$.

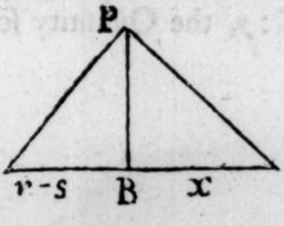
II. If $\frac{ab+cmn}{b+c} = x$. The Construction and Calculus will be more difficult; because no Letter in the Numerator is taken twice. But then it may be suppos'd that (n) for instance, may be reduc'd to [illegible] as $a : m :: n :$ a fourth Proportional which we'll call p. Then, $mn = ap$, and consequently $\frac{ab+ap}{b+c} = x$. Wherefore as by Rule [illegible] $b+c : b+p :: a : x$.

If this Equation were propoſed $\frac{ab+mn}{r-s}=x$.



You may find firſt a middle Proportional between *a* and *b*, which ſuppoſe to be *p*. Alſo another mean Proportional between *m* and *n*, which let be *q*. Then will the Equation ſtand thus, $\frac{pp+qq}{r-s}=x$.

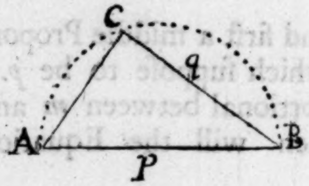
Let therefore a Right-angled Triangle be made, wherein the Perpendicular AP $=p$, and the Baſe AB $=q$. Therefore ſhall PB $q=pp+qq$, which, ſince according to the Equation it is to be divided by $r-s$: Make, as $r-s$ to BP ($=\sqrt{:pp+qq}$) :: ſo PB to a third Proportional; which ſhall be x ſought.



III. In this Equation $\frac{ab - mn}{c + d} = y$.

Firſt make, as $a : m :: n :$ (4th Proportional) which let be p. Then will $\frac{ab - ap}{c + d} = y$. And conſequently, (as in *Caſe* 1.) $c + d : b - p :: a : y$.

Or you might, as in (*Caſe* 2.) have found a mean Proportional between a and b, as alſo between m and n, which being called (as there) p and q. The Equation would have ſtood thus, $\frac{pp - qq}{c + d} = y$. Then having taken $AB = p$, and on it as a Diameter drawn a Semicircle; and applying in $BC = q$. The □ of $AC = pp - qq$. And conſequently $AC = \sqrt{:pp - qq}$; which ſince it is to be divided by $c + d$, make as $c + d : AC :: AC : y$, the Quantity ſought.



IV. Let this Eqhation $\frac{aabc}{ffg} = z$, be propoſed.

Firſt, Find oht (p) a 3d Proportional to f and a, Then fp being $= aa$. The Equation will ſtand thus, $\frac{fpbc}{ffg} = z$, (i. e.) $\frac{pbc}{fg} = z$.

Secondly, Find a fourth Proportional (q) to f, p, and b, ſaying, as $f : p :: b : q$. Then will $fq = pb$, and conſequently the Equation will ſtand thus, $\frac{fq}{fg} = z$ (i. e.) $\frac{qc}{g} = z$. And therefore, (as by *Numb.* 1.) $g : q :: c : z$ ſought.

V. If this Equation $\frac{bkk}{mm} = x$ were propoſed.

Firſt, Find a fourth Peoportiohal to m, b and k, which let be p, therefore $pm = bk$, and conſequently the Equation will ſtand thus, $\frac{pmk}{mm} = x = \frac{pk}{m}$: Therefore $m : p :: k : x$ ſought.

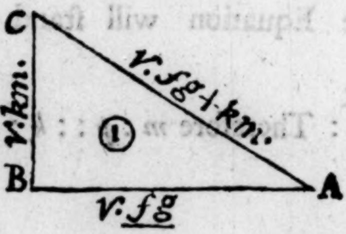
Conſtruction of Simple Quadratick Equations.

N. B, *Simple Quadraticks, are ſuch as have on one ſide of the Equation only ſome Power of the Quantity ſought, without any other Quantity or Letter mixt with it.*

I. IF an Equation be in any of theſe following or the like Forms.

$$\left.\begin{array}{l} yy=ab \\ yy=1c \text{ or } c \\ yy=\frac{3}{4}dd \end{array}\right\} \text{therefore} \left\{\begin{array}{l} y=\sqrt{ab} \\ y=\sqrt{c} \\ y=\sqrt{\tfrac{3}{4}dd} \end{array}\right\} \begin{array}{l} \text{that is } y= \\ \text{to a mean} \\ \text{proportional be-} \\ \text{tween} \end{array} \left\{\begin{array}{l} a \,\&\, b \\ 1 \,\&\, c \\ \frac{3}{4}d \,\&\, d \end{array}\right.$$

II. If this Quadratick be propoſed; $yy=fg \pm km$ Then will $y=\sqrt{:fg \pm km}$.



1. When the Sign is + make a Rectangled Triangle A B C, and let the Side A B = to a middle Proportional between *f* and *g*, and the the Side C B = to a mean Proportional between *k* and *m*.

2. But

[illegible] Part of the [illegible] corresponding Scheme [illegible] Hypothenuse [illegible]

is the mean Proportional between y and [illegible], and the Side B C [illegible] mean Proportional between [illegible] and m, [illegible] thing is done.

III. If such an Equation as this be propos'd, viz. [illegible] $= xx$. There will x be [illegible]

and by Extracting the Square Root in [illegible] (because all the Quantities are Simple and Perfect [illegible]) [illegible] x will be [illegible]. Therefore [illegible]

[illegible] Simple Equation.

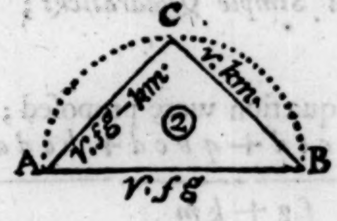
IV. If this Equation be propos'd [illegible]

First find a Fourth Proportional to c, f and g, which let be p. [illegible]

[illegible]

Again, find a Fourth Proportional to [illegible] and [illegible], which let be q. [illegible] Therefore [illegible]

2. Bht if the Sign be —, make the Rectangled Triangle (N. 2. whoſe Hypothenhſe A B let be =



to the mean Proportional between f and g, and the Side B C = to a mean Proportional between k and m, and the thing is plain.

III. If ſuch an Equation as this be propoſed, *viz.* $\frac{fhhdd}{fcc} = xx$. Then will it be $\frac{hhdd}{cc} = xx$, and by Extracting the Square Root every where; (becauſe all the Quantities are Simple and Perfect Squares) it will be $\frac{hd}{c} = x$. Therefore $c : h :: d : x$. As in *Caſe* 1. of *Simple Equations.*

IV. If this Equation be propos'd, $\frac{fghk}{dm} = xx$.

Find firſt a fourth Proportional to d, f and g, which let be p. Then $dp = fg$, and conſequently $\frac{dphk}{dm} = xx$. (*i. e.*) $\frac{phk}{m} = xx$.

Agein, find a fourth Proportional to m, p and h, which let be q. Then $qm = ph$, and therefore

$\frac{qmk}{m} = xx$, (i. e.) $qk = xx$. Which brings it to *Caſe* I. of *Simple Quadraticks*; which ſee, &c.

V. If this Equation were propoſed;

$$\frac{qbcc + bccd + qbcd + bcdd}{fg + km} = xx.$$

Firſt, Reduce the Rectangles fg and km to the Squares bb and qq by finding b and q mean Proportionals between f and g, and k and m, and make the Square $nn = bb + qq$.

Then will $\frac{qbcc + bccd + qbcd + bcdd}{nn} = xx$.

Alſo, becauſe cc and cd are Multiplied into $qb + bd$ find a Square = to the two Rectangles $qb + bd$, and let it be pp. Therefore $\frac{ppcc + ppcd}{nn} = xx$.

Again, Becauſe pp is multiplied both into cc and cd, find another Square equal to $cc + cd$, which let be qq. Then will $\frac{ppqq}{nn} = xx$ (i. e.) $\frac{pq}{n} = x$. Therefore $n : p :: q : x$ ſought.

Queſtions producing Simple Equations.

QUeſtion I. 200 l. *is to be divided between two Men, that one is to have* 73 l. *more than the other: What is the Share of each?*

For the whole Money put $s = 200$ *l.*

For the Difference between their Shares, put $d = 73$ *l.*

And let the Shares ſought be a and e?

$$\begin{array}{r|c|l} & \begin{array}{c} 1 \\ 2 \end{array} & \left.\begin{array}{l} a+e=s \\ a-e=d \end{array}\right\}\text{by the Queſtion.} \\ 1+2 & 3 & 2a=s+d. \end{array}$$

N. B. *'Tis an Excellent way to keep a Regiſter in the Margin of all the Steps in the Reſolution of any Equation (according to Dr.* Pell*'s Method;) for by that means it will readily appear how every Step is produced: As here the Figures* 1 + 2 *right againſt the third Step ſhew you that it is produced by adding the firſt and ſecond Steps together, which is done to deſtroy* e, *that* a *may ſtand alone.*

$$\begin{array}{r|c|l} 3 \div 2 & 4 & a=\dfrac{s+d}{2}: \left\{\begin{array}{l}\text{By dividing the laſt Step} \\ \text{by 2.}\end{array}\right. \end{array}$$

And thus is the Value of *a* preſently known, (*viz.* = 136 *l.* 10 *s.*) and this Theorem alſo gain'd, *That half the Sum added to half the Difference of a-*

ny two Quantities, is always equal to the greater of them: Which is the Senſe, in Words at length, of

$$\frac{s+d}{2} \text{ (or } \tfrac{1}{2}s+\tfrac{1}{2}d) = a.$$

And as this way you found the Value of *a* by Addition of the firſt and ſecond Steps; ſo you may find *e* by Subtraction of the ſecond Step from the firſt. Thus,

$1-2 \mid 5 \mid 2e = s - d$: Wherefore,

$5 \div 2 \mid 6 \mid e = \dfrac{s-d}{2}$

That is in Words, *Half the Sum of any two Quantities leſs half their Difference is always equal to the leſſer of them:* Which is a Canon or Theorem that will find *e* to be 63 *l.* 10 *s.*

So that having the Sum and Difference of any two Quantities or Numbers, 'tis, you ſee, very eaſie to diſcover the Numbers themſelves.

Queſtion II. 200 l. *is ſo diſtributed between two Men, that if the Share of one be divided by that of the other, the Quotient will be* 3. *How much had each?*

For the Shares, put *a* and *e*;
Then $a + e = s = 200.$

$$\frac{a}{e} = q = 3.$$

The first Question may be answer'd thus,

1 $a + e = s = 200$
2 $a - e = d = 73$
1−2 3 $2e = s - d = 127$
3÷2 4 $e = 63\frac{1}{2}$
1−4 5 $a = 136\frac{1}{2}$ which thing requir'd

or thus

for ye greatest share put a, then by ye Question, the least share is $a - d$

1 $a + a - d = s$
2 $2a = s + d$
3 $a = \frac{s+d}{2}$

as before.

The 2d Question may be answer'd thus,

1 $a + e = s = 200$
2 $\frac{a}{e} = q = 3$
2×e 3 $a = 3e$
1,3, 4 $4e = 200$
4÷4 5 $e = 50$
1−5 6 $a = 150$

Or thus

for the least share put — E
then by ye Question the greatest share = 3E

1 $3E + E$, or $4E$, $= S = 200$
1÷4 2 $E = \frac{S}{4} = 50$
E×3 3 $3E = 150$ –

Queſtions producing ſimple Equations.

What numbers are thoſe, from ye greater of which if 1 be taken, & added to ye leſser, both will be equal: but if to ye greater 1 taken from ye leſser be added 'twill be twice as much as the leſser.

Anſw: 7 & 5

Let $a = 7$
$e = 5$
$b = 1$

	1	$a - b = e + b$
	2	$a + b = 2e - 2b$
2−1	3	$2b = e - 3b$
Tranſ:	4	$5b = e$
	5	& conſequently $7b = a$.

	1	$e = s - a$
	2	And ſince $\frac{a}{e} = q$. If you Multiply both by $e : a = eq$.
2 : q	3	$\frac{a}{q} = e$, by which means e vaniſhes, being expreſſed another way.
1, 3,	4	$s - a = \frac{a}{q}$ by conſidering the firſt and ſecond Steps; (which is always implyed by the Comma's in the Margin.)
4 × q	5	$sq - qa = a$, Multiplying both by q.
Tranſp.	6	$sq = a + qa$, by Transferring qa.
6 : $1+q$	7	$\frac{sq}{1+q} = a$, by Dividing each part by the Coefficient $1 + q$; that is, $a = 150\ l$.
	8	$e = 50\ l.$ becauſe a was 3 times as much as e by the Queſtion.

If inſtead of the Sum of the 2 Shares, the Difference $d = 100$ had been given, as alſo the Quotient $\frac{a}{e} = 3 = q$; and a and e required?

Then,

	1	$e + d = a$.
	2	$a = eq$ { by the ſecond Step of the former Queſtion.
1, 2,	3	$e + d = eq$.
Tranſp.	4	$d = eq - e$.
$4 : q - 1$	5	$\frac{d}{q-1} = e = 50\,l.$ Wherefore,
		$a = 3e = 150\,l. = d + e$.

Queſtion III. *Two Men have between them* 56 l. *and the greater Share is to the leſſer as* 5 *to* 2, *or as r to t. What had each?*

Put *a* and *e* for the 2 Shares, and $s = 56\,l.$

	1	$s = a + e$.
	2	$r : t :: a : \frac{ta}{r} = e$.

This Step ariſes only by ſaying according to the *Golden Rule*; if *r* give *t*, What ſhall *a* give? The fourth Term is $\frac{ta}{r} = e$. which finds a new Notation for *e*.

Three men have a certain ſum ſo divided between them that the ſhare of the firſt is to the ſhare of y^e 2^d as 4 to 3 and exceeds it by 6 y^e ſhare of y^e ſecond is to that of y^e 3^d as 9 to 7 & y^e whole ſum is 56.

Queſt: how much is the ſhare of each?

Let y^e ſhare of y^e firſt = a

y^e ſhare of y^e ſecond = u

y^e ſhare of y^e third = y

4 = b
3 = c
9 = d
7 = g
56 = h
6 = k

	1	$a - \frac{ca}{b} = k$
2×b	2	$ab - ca = kb$
2÷b−c	3	$a = \frac{kb}{b-c} = 24$
	4	$a - u = 6$
~~[illegible]~~ 3−4	5	$u = 18$
	6	$h = a + u + y = 56$
3+5	7	$a + u = 42$
6−7	8	$y = 14$

Queſtion 3 may be anſwer'd thus.

Let y^e greateſt Share = a

	1	Then, by y^e Queſtion, $5 : 2 :: a : \frac{2a}{5}$ = y^e leaſt Share
	2	$a + \frac{2a}{5} = ſ$
2×5-	3	$5a + 2a\ [=7a] = 5ſ$.
3÷7	4	$a = \frac{5ſ}{7} = \frac{280}{7} = 40$

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1, 2,	3	$a+\frac{ta}{r}=S.$
$3\times r$	4	$ra+ta=rS.$
$4\div r+t$	5	$a=\frac{sr}{r+t}=\frac{280}{7}=40\,l.$
		And conſequently,
1, 5,	6	$e=\frac{80}{5}=16\,l.$

If inſtead of the Sum, the Difference $d=$ ~~16.~~ 24 had been given. Then,

	1	$a-\frac{ta}{r}=d\left\{\begin{array}{l}\text{by working for } e \text{ as}\\ \text{in Step 2 of the laſt.}\end{array}\right.$
$1\times r$	2	$ra-ta=rd$
$2\div r-t$	3	$a=\frac{rd}{r-t}=\frac{120}{3}=40$
	4	$e=a-d=40-24=16$, as before.

Queſtion IV. *One having a certain Number of Eggs, left (without breaking any) half that Number and half an Egg at one Place: Half the Remainder and half an Egg at a ſecond place: Half the Remainder and half an Egg at a third Place: And then he had one Egg left: How many had he at firſt?*

For the Number of Eggs put *a* ?
And for one Egg put $1 = b$.

1 | $\frac{a}{2} + \frac{b}{2} =$ { what he left at the firſt Place. And,

2 | $\frac{a}{2} - \frac{b}{2}$ or $\frac{a-b}{2} =$ { to the firſt Remainder.

And if to avoid the trouble of *Vulgar*, you would expreſs it by *Decimal* Fractions: Then the firſt Eggs left will be $\frac{a}{2} + :5$ and the firſt Remainder $\frac{a}{2} - :5$.

3 | $\frac{a}{4} - \frac{b}{4} + \frac{b}{2}$: or $\frac{a}{4} - :25 + :5 =$ to what he left at the ſecond place: And,

4 | $\frac{a}{4} - \frac{b}{4} - \frac{b}{2}$: or $\frac{a}{4} - :25 - :5$ will be the ſecond Remainder.

Tis said that a certain man went to an alehouse, & having borrow'd there as much mony as he had in his pocket spent 6^{d}, & then went to another house, & did the like, & when he had done y^{e} same at a 3^{d} place, he had no money left.

Quest: How much mony had he at first?

	1	$a =$ y^{e} mony he had at first.
	2	$b = 6^{d}$.
$2a-b$	3	$2a-b =$ what he had left when he went from y^{e} first house.
$2\times3-b$	4	$4a-2b-b =$ what he had left whn he went from y^{e} 2^{d} place.
$2\times4-b$	5	$8a-4b-2b-b = 0$ for he had nothing left when he went from y^{e} 3^{d} place.
trans:	6	$8a = 7b = 42$
$6 \div 8$	7	$a = \frac{7b}{8} = 5\frac{1}{4}$ & so much mony he had at first.

Suppose B & D run a race that contains 40000 of B's ſteps, & that as often as B makes 7 ſteps D makes 6, but 4 of D's ſteps are as much as 5 of B's, alſo that B has run 2560 of his own ſteps before D ſet out; which of them will win ye race, & how much will he be before the other when he comes to the Goal?

1 Let $40000 = F$
2 $7 = n$
3 $6 = m$
4 $4 = r$
5 $5 = s$
6 $2560 = t$
& ye number of ſteps wch D muſt run before he overtake B
7 $= a$

8 $m : n :: a : \frac{an}{m}$

9 $\frac{an}{m} + t =$ all B's ſteps wh D overtakes him

9=10 10 $\frac{an+tm}{m} = \frac{an}{m} + t$

11 $r : s :: a : \frac{an+tm}{m}$

12 $sa = \frac{ran+rtm}{m}$

12×m. 13 $sam = ran + rtm$

trans: 14 $sam - ram = rtm$

$14 \div \frac{sm}{-rm}$ 15 $a = \frac{rtm}{sm-rm} = \frac{10240}{6} = 1706\frac{2}{3}$

16 $n : m :: F : \frac{Fm}{n}$ = all D's ſteps = $(34285\frac{5}{7}$

16−15 17 $\frac{Fm}{n} - a = 32579\frac{1}{21}$ so that when D has $32579\frac{1}{21}$ ſteps to run when he overtakes B.

see the leaf next to page ye laſt.

5 $\frac{a}{8}-\frac{b}{8}-\frac{b}{4}+\frac{b}{2}$, or $\frac{a}{8}-{:}125-{:}25+{:}5=$, to what he left at the third Place.

6 $\frac{a}{8}-\frac{b}{8}-\frac{b}{4}-\frac{b}{2}$, or $\frac{a}{8}-{:}125-{:}25-{:}5=$, the third Remainder; which by the Queſtion is $=$ to b or 1. Therefore,

7 $\frac{a}{8}-\frac{b}{8}-\frac{b}{4}-\frac{b}{2}=b$, or $\frac{a}{8}-{:}125-{:}25-{:}5=1$. Wherefore,

Tranſp. 8 $\frac{a}{8}=b+\frac{b}{8}+\frac{b}{4}+\frac{b}{2}$, or $\frac{a}{8}=1+{:}125+{:}25+{:}5$, or to $1{:}875$.

$8\times\overline{8}$ 9 $a=8b+b+\frac{8b}{4}+\frac{8b}{2}$, or $a=15{:}000$. Wherefore $a=15$.

So that the Import of the Queſtion is only to find a Number, from which taking half and half an Unite; and from the Remainder its half and half an Unit: The laſt Remainderr ſhall be $=$ to 1. 'Tis plain alſo, That if a fourth time he had left half the Remainder and half an Egg, he would have had nothing left.

Qneſtion V.

Acer in Æmonia fugientem valle Lyciſca
Inſequitur Leporem, picta per arva, vagum:
Hic decies quinis præcedit paſſibus, ille
Inſtat, & exultans per Juga læta ruit:
Dumq; Quater ſaliendo Lepus conſurgit in altum
Hic Toties Ternis Saltibus evehitur.
At tantum geminis percurrit Saltibus Agri
Interea, Quantum conficit ille Tribus.
Dic mihi jam, Quoties ſaltus Iterante Lyciſcâ,
Contigit Infeſto præda petita cani?

Clark's Ought. explicat.

A Hare being 50 *Paces before a Greyhound, makes four Leaps to the Dog's three; but two Leaps of the Dog's are as much as three of the Hare's. How many Leaps muſt the Greyhound take to catch the Hare?*

$$\text{Let } 50 = b.$$
$$4 : 3 :: r : s$$
$$2 : 3 :: m : n.$$

For the Number of the Dog's Leaps ſought, put a.

$$\left|\, 1 \,\right|\; s : r :: a : \frac{ra}{s}, \text{ \&c.}$$

Say, As the Number of the Dog's Leaps, to thoſe of the Hare in any time: So will all the Dog's way be to all the Hare's, after he began to Courſe her.

	2	$\frac{ra}{s}+b$, or $\frac{ra+bs}{s}=$ the whole Number of the Paces the Hare went.
	3	$m:n::a:\frac{sb+ra}{s}$

This Proportion ſhews you, that as 2 is to 3, ſo is the whole Number of the Dog's Leaps to that of the Hare's.

	4	$na=\frac{sbm+mra}{s}$ becauſe in four Proportionals, the Rectangle of the Extreams is = to that of the mean Terms.
4 × s	5	$sna=sbm+mra$.
Tranſp.	6	$sna-mra=sbm$.
6 ÷ $sn-mr$	7	$a=\frac{sbm}{sn-mr}=300$.

Queſtion VI. *In three Bags there is a certain Quantity of Pounds Sterling. The Sum of the Pounds in the firſt and ſecond Bag is* 20 l. *The Sum of the Pounds in the ſecond and third Bag is* 48 l. *And the Sum of the Pounds in the firſt and third Bags is* 44 l. *What Number of Pounds was in each?*

Let *a e* and *y* be put for the Quantities ſought: And 20 = *b* : 48 = *c* and 44 *d*.

Then by the Queſtion.

$$\begin{cases} a+e=b \\ e+y=c \\ y+a=d \end{cases}$$ 1, 2, 3

4 | $e=b-a$, by Tranſpoſition of a.

5 | $y+b-a=c$, $\left\{\text{becauſe } b-a=e, \text{ and } e+y=c.\right.$

6 | $y=a-b+c$, $\left\{\text{by Tranſpoſition of } b-a.\right.$

,6, | 7 | $a-b+c+a=d$, $\left\{\text{That is } 2a-b+c=d.\right.$

Therefore,

Tranſp. | 8 | $2a=d+b-c$.

9 | $a=\dfrac{d+b-c}{2}=8$ $\left\{\text{Which is a Canon to find } a.\right.$

4, | 10 | Then ſince $b-a=e$: $e=12$.

6, | 11 | And $y=36$.

This is called a Queſtion by various Poſition, of which you have many in *Kerſey*, and other Writers.

PROBLEM I.

To determine the Point where a Line perpendicularly let fall from the Vertex or top of an Acute-angled Triangle, d b c *ſhall cut the Baſe* b.

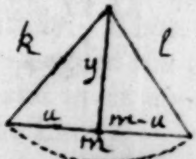
Suppoſe it done, and the Figure drawn, call one Segment of the Baſe a; then will the other be $b-a$, and let the Perpendicular be called p.

This Question
may also be answer'd thus,

1 | $a+e=b=20$
2 | $e+y=c=48$
3 | $y+a=d=44$
2−3 | 4 | $e-a=c-d=4$ thus y is destroy'd
1−4 | 5 | $2a=16$
5÷2 | 6 | $a=8$
4+6 | 7 | $e=12$
2−7 | 8 | $y=36$. which is a much shorter way than the other.

Problem 1st

Let yͤ Triangle be klm, let k be $=5$ $l=5$ $m=6$



1 | $yy=kk-uu$. & $yy=ll-mm+2mu-uu$
Trans: 2 | $kk=ll-mm+2mu$.

Transp:	3	& conseq. $kk + mm - ll = 2mu$
$3 \div 2m$	4	And therefore $\frac{kk+mm-ll}{2m} = u$
	5	& $u = 3$

Let there be sought 2 Square Numbers, in ye same proportion as 9 to 4, whose Roots may be express'd by real Numbers. Call ye Less Number a^2, & ye greater a^4. Then

	1	$9 : 4 :: a^4 : a^2$
by 19, of 6th of Eucl.	2	$9a^2 = 4a^4$
$2 \div$ by a^2	3	$9 = 4a^2$
$3 \div$ by 4	4	$2\frac{1}{4} = a^2$
$\sqrt{}$ of 4th.	5	$1\frac{1}{2} = a = \sqrt{}$ Root of ye lesser Number
Inversion	6	$4 : 9 :: 2\frac{1}{4} : 5\frac{1}{16} = a^4$. the $\sqrt{}$ of wch is $2\frac{1}{4}$. & so two Square Numbers are found in ye proportion of 9 to 4. Q. E. F.

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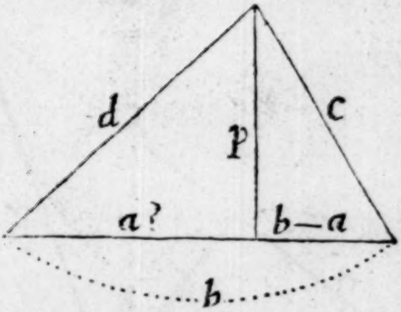
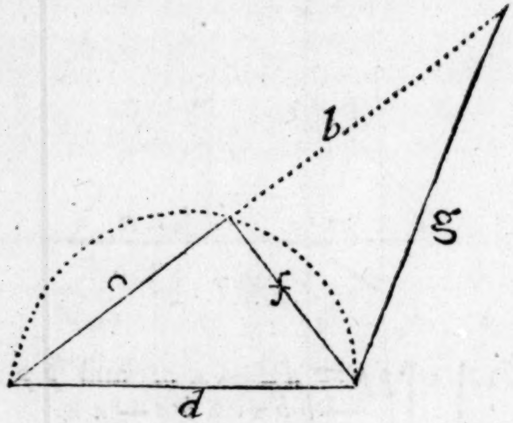


47. c. 1.	1	$pp = dd - aa$, and $pp = cc - bb + 2ba - aa$.
Tranſ. aa	2	Wherefore $dd = cc - bb + 2ba$.
Tranſp.	3	And conſeq. $dd + bb - cc = 2ba$.
3 -: $2b$	4	And therefore $\dfrac{dd + bb - cc}{2b} = a$.

Wherefore the length of *a* is found Arithmetically, or by Calculation; for if you add the Square of the Baſe *b*, and of the Side *d* (Meaſured on any Scale) together, from the Sum Subtract the Square of the Side *c*: And then divide the Remainder by the double of the Baſe *b*, the Quotient will give you the length of *a*, and conſequently determine the Point where *p* will cut the Baſe *b*.

And you may conſtruct the Equation in the fourth Step Geometrically. Thus,

On *d*, the longeſt Leg of the given Triangle make a Semicircle; and in it apply *c*, the other Leg, drawing alſo the Line *f*.



Then will $ff = dd - cc$, produce c till the Part without the Semicircle be equal to b, the Baſe of the given Triangle : And draw the Line g.

Then will $gg = ff + bb = dd - cc + bb$, and ſince in the fourth Step, this laſt Quantity is to be divided by $3\,b$, make, as $2\,b : g$, (which is $=$ to $\sqrt{:\overline{dd - cc + bb}}$) :: g to a fourth Proportional, which will be a. And conſequently a will be found Geometrically.

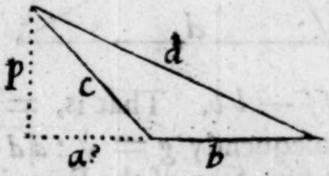
PROBLEM

[illegible]

[illegible]

PROBLEM II.

To find the Point without an Obtuſe-angled Triangle d c b, *where the Baſe* b *being produced, ſhall meet with a true Perpendicular* p *let fall from the Vertex of the Triangle.*



Suppoſe it done, and all things noted as you ſee in the Figure. Therefore,

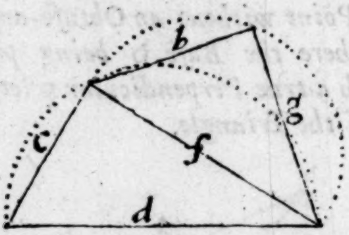
47. *e.* 1.	1	$pp = cc - aa$, and $\overset{p}{p_{\wedge}} = dd - aa - 2ba - bb.$
Tranſp. *aa*	2	$cc = dd - 2ba - bb.$ And,
By Tranſp.	3	$cc + 2ba = dd - bb.$ And,
	4	$2ba = dd - bb - cc.$ Wherefore,
5 −: 2 *b.*	5	$a = \dfrac{dd - bb - cc}{2b}.$

And conſequently *a* is found by Calculation.

The Geometrical Conſtruction.

On *d* the longeſt Leg of the Triangle given, deſcribe a Semicircle, and in it apply *e* the other Leg: Drawing alſo the Line *f*, ſo will $ff = dd - cc$: Then on *f* deſcribe alſo another Semicircle,

circle, and therein apply b the Baſe of the Triangle given: Drawing likewiſe the Line g. Then



will $gg = ff - bb$. That is, $= \overline{dd - cc - bb}$: And conſequently $g = \surd : \overline{dd - cc - bb}$. And ſince the fifth Step of the Equation is divided by $2b$; make, as $2b$ to g :: ſo g to a fourth Term; which will be a, the Side ſought.

Quadratick Equations.

QUadratick Equations, are ſuch as retain on the unknown Side, the Square of the Root or Number ſought; and are of two ſorts.

I. *Simple Quadraticks*, where the Square of the unknown Root is equal to the abſolute Number given, as $aa = 36$, $ee = 146$, $yy = 133225$. And for the Solution of theſe, there needs only to Extract the Square Root out of the known Number, and that is the Value of the Root or Quantity ſought: Thus the Value of a in the firſt Equation is equal to 6, in the Second $e = 12$ and a little

little more, it being a Surd Root. And in the Third Example $y - 365$.

II. *Adfected Quadraticks*, are ſuch as have between the higheſt Power of the unknown Number, and the abſolute Number given, ſome intermediate Power of the unknown Number, as $aa + 2ba = 100$.

And this Equation is properly called *Adfected*, becauſe the unknown Root a is Multiplied into the Coefficient $2b$.

The Original of *Adfected Equations*, the Incomparable *Harriot* thus derives: Let a be $= +b$, or $a = -c$, then by Tranſpoſition will $a - b = 0$, and $a + c = 0$. And then Multiplying one by another, the Product is $aa - ab + ca - bc = 0$.

And this he properly calls an *Original Equation*. From which, or others of the ſame kind, Tranſpoſing bc over to the other ſide with a contrary Sign, he gains ſuch an Equation as this, $aa - ab + ca = bc$, which he calls a *Canonical Equation*.

And from hence, by putting Examples in all Caſes he ſhews, That every poſſible *Quadratick Equation*, hath *two Real Roots*, according to the Dimenſions of the higheſt Power; as being made up by the Multiplication of two Simple Equations. And that theſe two Roots may be either both Affirmative, or both Negative, and that ſometimes they are equal to each other, and ſometimes not, And from hence he finds, That the Abſolute Number bc, is always the Rectangle of the two Roots b and c, (or of the two Values of a:) And that if it have a Poſitive Sign, the two Roots have like Signs, but if a Negative one, unlike.

And, That the Coefficient of the middle Term is always the *Aggregate of both the Roots with contrary Signs*; and conſequently their *Difference*, when without its Sign. See more in his *Second Section*, and in *Wallis*'s *Algebra*, p. 132, &c.

And when in ſuch kind of Quadraticks as theſe, the Index or Exponents of the Dimenſions of the unknown Root are in Arithmetical Proportion; that is, as in this Equation, $aa + 2ba = 100$, the Index of aa is 2, the Index of $2ba$ is 1, and the Index of 100 is 0; then may the Root be eaſily found by the following Method.

All Equations of this Rank will be in one of theſe three Forms.

$$\left.\begin{array}{l} aa + ad = \mathrm{R}. \\ aa - ad = \mathrm{R}. \\ as - aa = \mathrm{R}. \end{array}\right\}$$

* *Some make four Forms, but at at long run it comes to the ſame thing.*

In all which Forms, R, the abſolute Number given, is a Rectangle or Product made out of the two Quantities or Roots ſought, a Greater and a Leſſer.

Of which, in the *Firſt Form*, where all is Affirmative, the Coefficient d is the Difference between thoſe two Quantities or Roots, and a is the Leſſer of them; as is plain, if you ſuppoſe the two Roots (as *Oughtred* doth) to be a the Greater, and e the Leſſer. For then let $d = x$ be the Difference between them; ſo that $e + x = a$, if then you Multiply each Part by e, it will be $ee + ex = ae$; from whence it appears alſo plainly, that ae is equal to R, the abſolute Number given, or equal to the Rectangle of the two unknown

Roots

Roots a and e, of which in this Form, the Coefficient x or d is equal to the Difference between them, and e is the Leſſer of them.

In the *Second Form,* The Coefficient d is the Difference of the two Roots as before, but *a there* repreſents the Greater of them, as is plain, by putting (becauſe the Sign is Negative) $a - x = e$, and Multiplying each Part by a, it produces $aa - ax = ae$, the ſecond Form, where x or d the Coefficient is the Difference of the two unknown Roots; and a repreſents the Greater of them.

In the *Third Form,* where the higheſt Power is Negative, the Coefficient s is the Sum of the two Quantities or Roots ſought; and a the Affirmative Root ſought may be either the Bigger or the Leſſer of them. For let (becauſe the higheſt Power is Negative) $z - a = e$: Then Multiplying both by a, it will be $za - aa = ae = R$; or if $z - e$ had been put equal to a, then it would have been $ze - ee = ae$, by Multiplying all by e.

So that this Method ſhews the Original Conſtitution of theſe Forms, and the Nature and Office of each Member of them.

From all which may be found this General Canon for the Solution of *Quadratick Equations,* according to this Method.

Multiplying the abſolute Number by 4, and to the Product add the Square of the Coefficient, then Extract the Square Root of that Sum; which Root ſhall be the Sum of the two Numbers ſought. Then to or from the half of that Root, Add and Subtract half the Coefficient, and the Sum and Remainder are the two Roots required.

For the particular Solution of *Adfected Quadraticks*, there are three Ways.

I. *That of* Oughtred, *who proceeds in this Method.*

In all the three Forms, there is given either the *Rectangle* and *Sum*, or the *Rectangle* and *Difference* of the two unknown Quantities; whence 'tis very eaſie to find either the *Difference* in the former, or the *Sum* in the latter *Caſe*; and then having the *Sum* and *Difference* of any two unknown Quantities, the Quantities themſelves will ſoon be known.

Thus in the firſt Form. Let $aa + da = R$.

Here is given R, the Rectangle of the Roots, d their Difference; and 'tis known that a repreſents the Leſſer of them. Let S ſtand for the Sum to be ſought.

Let $a + e = s$, and $a - e = d$.
Then $aa + 2ae + ee = ss$, and $aa - 2ae + ee = dd$.
Subduct the latter from the former.

$$\begin{array}{r} aa + 2ae + ee \\ aa - 2ae + ee \\ \hline 4ae = 4R \end{array}$$

Wherefore $SS - dd = 4R$. And therefore,

You

[illegible] and ſubtract all the Coefficients, and the [illegible] and Remainder, [illegible] the two Roots [illegible]

[illegible] [illegible] Ways.

[illegible] of Quadraticks, [illegible]

Of all the three Forms, there [illegible] [illegible]

[illegible]

Thus in the [illegible] Form [illegible]

[illegible]

[illegible]

You may by *Simple Algebra*, find that $4R = SS - dd$, and conſequently that $4R + dd = SS$, and therefore S is known; and then having S and d, a the leſſer Root will be known too, for $\frac{1}{2}S - \frac{1}{2}d = a$.

Again, in the ſecond Form. Let $aa - ad = R$.

Here d and R (as before) the Difference and Rectangle of the two Roots are given; and a the greater of them; wherefore 'tis eaſie to find S the Sum, and then $\frac{1}{2}S + \frac{1}{2}d = a$.

In the third Form, where $Sa - aa = R$.

There is given the Coefficient S = Sum of the unknown Roots, R the Rectangle between them; and a may be either the bigger or the leſſer of them: Here therefore to find d the Difference.

Becauſe $SS - dd = 4R$, therefore $SS + 4R = dd$, and conſequently d is known; and then $\frac{1}{2}S + \frac{1}{2}d =$ greater, and $\frac{1}{2}S - \frac{1}{2}d =$ leſſer.

II. *The Solution of Adfected Quadratick Equations, by the Method of compleating the Square.*

Which is by Mr. *Harriot*, thus: Since in every one of the three Forms of *Quadraticks*, one Quarter of the Square of the Coefficient will make the unknown Side of the Equation, a compleat Square, whoſe true Root will be $a \mp \frac{1}{2}d$, (or whatever Letter elſe be the Coefficient.) 'Tis plain by this means, an *Adfected Quadratick Equation* may be reduced to a *Simple* one.

Wherefore,

In the firſt Form, when all the Species are Affirmative,

$$\text{Let } a\,a + d\,a = \text{R}.$$

If $\frac{1}{4}\,d\,d$ be added to the unknown Side, it will be a perfect Square $a\,a + d\,a + \frac{1}{4}\,d\,d$, whoſe true Root is $a + \frac{1}{2}\,d$.

Add then, $\frac{1}{4}\,d\,d$ to R, and $\text{R} + \frac{1}{4}\,d\,d$ will be a perfect Square Number and known; whoſe Square Root extracted in Numbers, will be equal to $a + \frac{1}{2}\,d$: And conſequently, a will be equal to that Root, when $\frac{1}{2}\,d$ is taken from it, and ſo a will be known.

The Practical Rule is thus.

To the Abſolute Number, add $\frac{1}{4}$ of the Square of the Coefficient, (or the Square of half the Coefficient) and Extract the Root of them: Then from that Root found in Numbers, Subtract $\frac{1}{2}$ the Coefficient, and the Remainder is a, the leſſer of the two Roots, or Values of a.

EXAM-

Examples

$aa + ba = G.$ or $aa + 1a = 12$

In y^e first form.

$\text{To } 12 = G$

$\text{Add } \frac{1}{4} = \frac{1}{4}bb$

$\sqrt[2]{}: 12\frac{1}{4} = 3\frac{1}{2} = a + \frac{1}{2}b$

$\text{But } \frac{1}{2}b = \frac{1}{2}$

Therefore $3 = a.$

In the 2^d form.

$yy - ny = D = 18$

$\text{to } 18 = D$

$\text{add } 2\frac{1}{4} = \frac{1}{4}nn$

$\sqrt[2]{}\, 20\frac{1}{4} = 4\frac{1}{2} = y - \frac{1}{2}n$

$\text{but } \frac{1}{2}n = 1\frac{1}{2}$

therefore $6 = y.$

In y^e 3^d form.

$Pu - uu = M = 10$

$uu - Pu = -M.$

$\text{From } 12\frac{1}{4} = \frac{1}{4}PP$

$\text{Take } 10 = M.$

$\sqrt[2]{}\, 2\frac{1}{4} = 1\frac{1}{2}$

EXAMPLE.

$$aa + da = R$$

Or, $aa + 16a = 36$.

To $36 = R$.

Add $\underline{64} = \frac{1}{4}dd$.

$\sqrt[2]{:100} = 10 = a + \frac{1}{2}d$.

But $\frac{1}{2}d = 8$.

Therefore $2 = a$.

In the ſecond Form. Let $aa - da = R$.

Proceed in all reſpects as in the firſt Form, only you muſt at laſt add half the Coefficient to the Root extracted out of the Abſolute Number, inſtead of taking it from it, as before; becauſe here a repreſents the greater Root: And thus, If $aa - 16a = 36$, a will be found $=$ to 18.

In the third Form. Let $Sa - aa = R$.

Here becauſe the higheſt Power is Negative, 'tis impoſſible any ſuch Root can be found that will produce $-aa$; wherefore you muſt imagine all the Signs changed, and it will ſtand thus, $-Sa + aa = -R$; or putting the higheſt Power firſt, $aa - Sa = -R$.

In this Form, the Coefficient is the Sum of the two Roots, and a may be either of them.

And here the Abſolute Number is ſo determined, as that it cannot be greater than the Square of half the Coefficients: Wherefore,

The Practical Rule is this.

From the Square of half the Coefficient, take the Abſolute Number given; and Extract the Square Root of the Remainder; which Root either added to, or Subtracted from half the Coefficient, will give accordingly the greater or leſſer Value of a.

Thus, If $20a - aa = -36$

Or, $Sa - aa = -R$

From $100 = \frac{1}{4}SS$

Take $36 = R$

$\sqrt[2]{:64} = 8$

Now $10 + 8 = 18$ the greater Root,

And $10 - 8 = 2$ the leſſer Root.

III. *To Solve* Quadratick Adfected Equations, *by taking away the* Second Term.

In any of the three Forms, if the Coefficient have a Negative Sign, put $e + \frac{1}{2}d$; but if it have an Affirmative Sign, put $e - \frac{1}{2}d$, inſtead of a, the Root of the higheſt unknown Power.

Then

Now $3\frac{1}{2} + 1\frac{1}{2} = 5$ y^e^ greater root

And $3\frac{1}{2} - 1\frac{1}{2} = 2$ y^e^ leſser root.

Then will $ee \mp ed + \frac{1}{4}dd = aa$

Alſo $\mp ed \mp \frac{1}{2}dd = \mp da$.

And theſe two Quantities added together, muſt be equal to the Abſolute Number given; and the Equation will become a Simple one.

In the firſt Form, $aa + da = R$, or $aa + 16a = 36$.

Let $e - \frac{1}{2}d = a$.

Then will $ee - ed + \frac{1}{4}dd = aa$

And $ed - \frac{1}{2}dd = ad$

Which added together, make $ee - \frac{1}{4}dd = R$

Therefore $ee = R + \frac{1}{4}dd$

And conſequently, $e = \sqrt{R + \frac{1}{4}dd}$

But $e - \frac{1}{2}d = a$,

Therefore $e = a + \frac{1}{2}d$

Conſequently $a + \frac{1}{2}d = \sqrt{R + \frac{1}{4}dd}$

Wherefore $a = \sqrt{R + \frac{1}{4}dd} - \frac{1}{2}d$. *Q.E.D.*

And ſince $a + \frac{1}{2}d = \sqrt{R + \frac{1}{4}dd}$; If each part of the Equation be Squared, there will ariſe,

$$aa + ad + \frac{1}{4}dd = R + \frac{1}{4}dd.$$

Which is the other common Canon for Solving *Quadraticks*, by adding to each Part the Square of half the Coefficient, in order to compleat the Square.

In the ſecond Form, $aa - ad = R$.

Let $e + \frac{1}{2}d = a$.

Then is $ee + ed + \frac{1}{4}dd = aa$

And $-ed - \frac{1}{2}dd = -ad$

Theſe added make $ee - \frac{1}{4}dd = R$

Therefore $ee = R + \frac{1}{4}dd$.

And $e = \sqrt{R + \frac{1}{4}dd}$.

But $e + \frac{1}{2}d = a$

Therefore $e = a - \frac{1}{2}d$

And conſequently $a - \frac{1}{2}d = \sqrt{R + \frac{1}{4}dd}$

Wherefore $a = \sqrt{R + \frac{1}{4}dd} + \frac{1}{2}d$.

Q. E. D.

And ſince $a - \frac{1}{2}d = \sqrt{R + \frac{1}{4}dd}$; if each Side of the Equation be Squared, you will have,

$$aa - ad + \frac{1}{4}dd = R + \frac{1}{4}dd.$$

Which is the common Canon for Solving Equations, by compleating the Square.

In

[illegible] the Coefficient, in order to complete the Square.

In the second Term, [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Therefore [illegible]

And [illegible]

[illegible]

[illegible]

And consequently [illegible]

Wherefore [illegible]

[illegible]

[illegible] Square [illegible]

[illegible]

[illegible]

[illegible]

In the third Form, $d a - a a = \mathrm{R}$.

Which Form muſt be thus changed,

$$a a - d a = -\mathrm{R}.$$

Then make as before, $e + \frac{1}{2} d = a$

And then $e e + e d + \frac{1}{4} d d = a a$

And $\underline{-e d - \tfrac{1}{2} d d = -a d.}$

Whoſe Sum is $e e - \frac{1}{4} d d = -\mathrm{R}$

Then is $e e = \frac{1}{4} d d - \mathrm{R}$

And $e = \sqrt{: \tfrac{1}{4} d d - \mathrm{R}.}$

And ſince, $e + \frac{1}{2} d = a$

$$e = a - \tfrac{1}{2} d = (\sqrt{\tfrac{1}{2} d d - \mathrm{R}.})$$

Wherefore (becauſe there are two Poſitive Roots in this Form)

$$a = \sqrt{: \tfrac{1}{4} d d - \mathrm{R}} \pm \tfrac{1}{2} d.$$

But the Value of *a* is Ambiguous, and you muſt generally try both Roots before you can find which will Solve the Queſtion: Whereas in the other two Forms the firſt *a* found, will be that required.

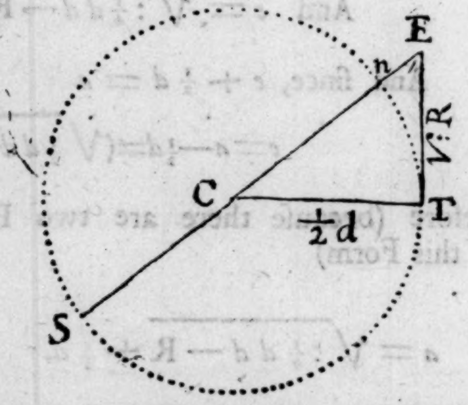
N. B. *In this way of Solving* Quadraticks, *the known Quantity added to, or Subtracted from* e, *muſt be always half the Coefficient.*

Conſtruction

Conſtruction of Adfected Quadraticks.

THE Conſtruction of *Simple Quadraticks*, you have before under *Simple Equations*: That of *Adfected* ones, is eaſily done many ways.

I. In the firſt Form of *Quadraticks*, let $aa + da = R$. Then by the common Method of Solution, $a = \sqrt{:R + \frac{dd}{4}} - \frac{d}{2}$.



Wherefore deſcribe a Circle whoſe Radius ſhall be C T $\frac{1}{2}d$, and make the Tangent T E $= \sqrt{:R}$, drawing alſo the Secant S C E; then will

$CE = \sqrt{:R + \frac{dd}{4}}$ (by 47. *e.* 1. *Euc.*) and conſeqnently $nE = \sqrt{R + \frac{d.d}{4}} - \frac{1}{2}d = a$.

'Tis requir'd to find two squares in y^{e} same proportion as 2 & 3 whose roots may be express'd by real numbers.

This ſeems to be impoſsible: but a Queſt. of y^{e} ſame nature ſee anſwer'd inter pag. 54^{am} & 55^{am}.

To prove this Queſtion an impoſsibility, let there be premis'd this

Lemma.

That y^{e} Quotient of any Square Number divided by another Square Number is a Square. For ſuch a Quotient ſtands thus $\frac{bb}{cc}$ w^{ch} 'tis plain is a Square Fraction.

Now call y^{e} 2 Numbers to be ſought $a^2 =<$ $e^2 =>$ Then

1 $2:3::a^2:e^2$ that is
2 $\frac{3}{2} = \frac{ee}{aa}$ or
3 $1.5 = \frac{ee}{aa}$ that is 1.5 w^{ch} is an Irrational Number $= \frac{ee}{aa}$ w^{ch} is Rational, w^{ch} is a Contradiction.

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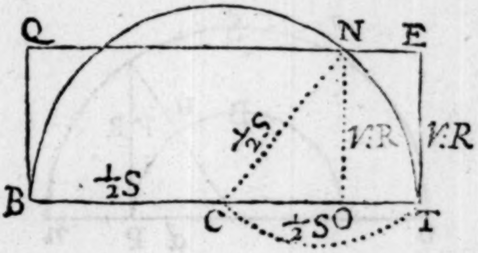
II. In the ſecond Form, where $aa - da = R$, a will be equal to $\sqrt{: R + \frac{dd}{4}} + \frac{d}{2}$. And conſequently, the ſame Conſtruction and Diagram will ſerve here, which was uſed in the firſt Form: And the Root will be repreſented by $SE = \sqrt{: \frac{1}{4}dd + R} + \frac{1}{2}d$.

III. In the third Form, where $Sa - aa = R$, a will be equal to $\frac{S}{2} + \sqrt{: \frac{SS}{4} - R}$, and here the Root a hath two real Values; make CT ($= \frac{1}{2}S$) the Radius of a Circle, and erect the Perpendicular $ET = \sqrt{: R}$; then draw EQ Parallel to CT, and NO Parallel to ET; draw alſo the Radius CN. Then will (by 47. *e.* 1 *Euclid.*) $CO = \sqrt{: \frac{SS}{4} - R}$, and conſequently, $BO = \frac{S}{2} + \sqrt{: \frac{SS}{4} - R} =$ the greater Root

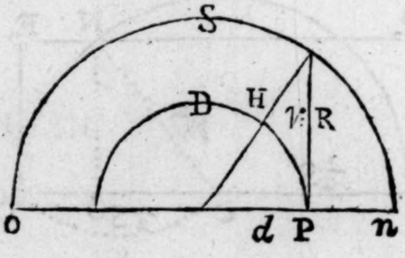


a,

a, and $O T = \frac{S}{2} - \sqrt{: \frac{SS}{4} - R}$. Or the two Roots will be Q N and N E, equal to the former.

Adfected Quadraticks.

DR. *Wallis*'s way of Conſtructing the three Forms of all *Quadratick Equations*, according to Mr. *Oughtred*'s Method of Solution. Draw two Concentrick Circles, and let the Diameter of the greater be called S, and the Diameter of the leſſer D, the Sum and Difference of the Roots found. Wherefore H and *d* will repreſent the half Sum and half Difference of the Roots.



Since therefore *Oughtred*'s *Theorem*, as is ſhewed above, is that $S S - D D = 4 R$. Let $\sqrt{R}$ be made a Tangent to the leſſer, or a Right Sine to the greater Circle, as you ſee in the Figure according as D, or S, is given: And draw alſo the Hypo-

Let 2 Numbers be found in a given Proportion, suppose yᵉ lesser to yᵉ greater as 2 to 3, & such yᵗ if yᵉ lesser Number be added to yᵉ Square of yᵗ greater; also if the greater number be added to yᵉ Square of yᵉ lesser, yᵉ 2 sums shall be square numbers whose roots are expressible by rational or true numbers (fractions being admitted for numbers)

Answ. $\frac{1}{10}$ & $\frac{3}{20}$.

What two numbers are those whose sum added to their squares = 72 but substracted from the product of y^e 2 numbers leaves 19?

For y^e sum of y^e numbers put 2a. & for their difference 2e: & then y^e numbers may be thus noted,

$a+e = >$ & $a-e = <$

1 | $2aa+2ee =$ Sum of their squares.

2 | $2aa+2ee+2a = 72$

2÷2 3 | $aa+ee+a = 36$

By trans: 4 | $36-aa-a = ee$

□-2a 5 | $aa-ee-2a = 19$

By trans: 6 | $aa-2a-19 = ee$

4.6. 7 | $36-aa-a = aa-2a-19$

By trans: 8 | $55 = 2aa-a$

8÷2 9 | $aa-\frac{1}{2}a = 27\frac{1}{2}$ which is a Quadratick of y^e 2^d form.

Compl: □ 10 | $27\frac{1}{2}+\frac{1}{16} = 27\frac{9}{16}$

11 | $\sqrt[2]{27\frac{9}{16}} = 5\frac{1}{4} = a-\frac{1}{4}$

By trans: 12 | $5\frac{1}{2} = a$

4,12, 13 | $36-30\frac{1}{4}-5\frac{1}{2} = ee = \frac{1}{4}$

14 | therefore $e = \frac{1}{2}$

15 | & $a+e = 6$ but

16 | $a-e = 5.$

Hypothenuſe H. Then will the Baſe of the Triangle be *d*. And $HH - dd = R$ (by 47. e 1.) That is, $\frac{SS}{4} - \frac{DD}{4} = R$. Wherefore by Tranſpoſition, $HH = R + dd$, and therefore $H = \sqrt{:R+dd}:$ And conſequently, if it had been in the firſt or ſecond Forms, where *d* and R were given, H will alſo be found. Or if H had been given, and *d* required as in the third Form. Since $HH = R + dd$; Therefore $HH - R = dd$: And $\sqrt{:HH-R} = d$: And having thus found H and *d*, the half Sum and half Difference of the two Roots. Then $H + d$ $(= op)$ will be the greater Root *a*, and $H - d$ $(= pn)$ will be the leſſer, which will be Affirmative or Negative, according to the Form and Circumſtances of the Equation.

A Queſtion and Problems in Adfected Quadratick Equations.

QUESTION.

Two Men have each a certain Number of Crowns, whoſe Sum Subtracted from the Sum of their Squares, leaves $R = 78$ *: But their Sum added to the Product of the two Numbers, makes* $39 = S$*. How many Crowns had each?*

For the unknown Sum of the Numbers put $2a$. And for their Difference $2e$: For then the Numbers may be thus noted, $a + e = >$ and $a - e = <$.

Where $>$ and $<$ ſignifie the Greater and Leſſer.

Then,

	1	$2aa + 2ee =$ Sum of their Squares.
$1 - \overline{2a}$	2	$2aa + 2ee - 2a = R$. by the State of the Queſtion.
$2 -: \bar{2}$	3	$aa + ee - a = \frac{R}{2} : 39 : = S.$
By Tranſ.	4	$39 - aa + a = ee$, which Step will at laſt find e.
$\{\square + 2a$	5	$aa - \overset{e}{\wedge} e + 2a = S$. Their Product added to their Sum.
By Tranſ.	6	$aa + 2a - S = ee.$
4, 6,	7	$39 - aa + a = aa + 2a - 39\ (S) = ee.$
By Tranſ.	8	$78 = 2aa + a.$
$8 : \bar{2}$	9	$aa + \frac{1}{2}a = 39 = S$; which is a Quadratick of the firſt Form.
Compl. $\square$	10	$aa + \frac{1}{2}a + \frac{1}{16} = 39 + \frac{1}{16}.$
ɯ	11	$a + \frac{1}{4} = \sqrt{:39 + \frac{1}{16}}.$
	12	$a = \sqrt{:39 + \frac{1}{16}} - \frac{1}{4} = 6.$
	13	Therefore $2a = 12.$

And (a) being known, the value of (e) will be found from the fourth Step. Where $e = 3$.

Now, by our Suppoſition at firſt, the greater Number was $a + e$, that is 9; and the leſſer was $a - e$; that is 3: Which Numbers 3 and 9, will Anſwer the Queſtion.

For

I ſuppoſe y^e 5th equation ſhou'd have been thus ≡ aa − ee + 2a = S their product added to their ſum.

The 7th ſhould doubtleſs have been written thus ≡ 39 (not 30) − aa &c.

What 2 numbers are theſe to y^e ſquare of y^e firſt of which if you add y^e other = 13; but if to y^e ſquare of the other the ~~ye~~ former be added = 19?

For y^e former number put a − e = <
for y^e latter a + e = >, then proceed thus,

1 | $aa - 2ae + ee =$ y^e ſquare of $a - e = <$
1 + a + e | 2 | $aa - 2ae + ee + a + e = 13$
By trans: | 3 | $13 - aa + 2ae - a - e = ee$

aa

the square of y^e latter + a−e.	4	$aa+2ae+ee+a-e=19$
Transp: ~~&~~ + 2ee.	5	$aa+2ae+2ee+a-e-19=ee$
3, 5,	6	$13-aa+2ae-a-e=aa+2ae+2ee+a-e-19$
Transp:	7	$32=2aa+4ae+2ee$
7÷2	8	$16=aa+2ae+ee$
4−2	9	$4ae-2e=6$
9÷2	10	$2ae-e=3$
$8 \longrightarrow 10$	11	$13=aa+ee+e$
2, 9.	12	$ee+e=-2ae+ee+a+e$
Transp:	13	$ee=-2ae+ee+a$
13 − ee	14	$0=-2ae+a$
Transp:	15	$2ae=a$
15÷a	16	$1=2e$
16÷2	17	$e=\frac{1}{2}$
a+e−a−e	18	$a+e-a+e=1$. having thus found the difference between y^e 2 numbers take new notations for them & instead of $a+e=>$ put
y^e new notatio y^e new notation.	19	$y=>$. instead of $a-e=<$ put $u=<$, & y^e ~~first~~ 2^d equation will stand thus,
	20	$yy+u=19$ & since you know $u+1=y$ instead of u put y & add 1 to 19
y put for u	21	$yy+y=20$. which is a Quadratick of the first form.
Comp: □	22	$\sqrt[2]{20\frac{1}{4}}=4\frac{1}{2}=y+\frac{1}{2}$ &
$22-\frac{1}{2}$	23	$y=4=a+e$
	24	$yy=16$
20−24	25	$u=3=a-e$

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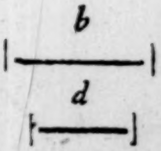
Or you may find the value of a+e & a−e without changing

For 12, their Sum, taken from 90, the Sum of their Squares, leaves 78; and added to 27, their Rectangle, makes 39.

N. B. *By this Method of putting* a + e and $a - e$ *for the two Numbers ſought, inſtead of* a *and* e, *as in the common way; many Queſtions producing Adfected Quadratick Equations, when that way manag'd, may be ſolved as eaſily, and in the manner of Simple Equations. Eſpecially when the Sum and Difference, or Sum or Difference of the Squares of the Quantities ſought, are among the* Data.

PROBLEM I.

The Difference of both the Legs of a Right-angled Triangle being given from the Hypothenuſe; to find the Sides and the Triangle ſeverally, and to Form it.



Let the Difference of the leſſer Side from the Hypothenuſe be (*b*) and that of the greater (*d.*) For the greater Side ſought put (*a.*)

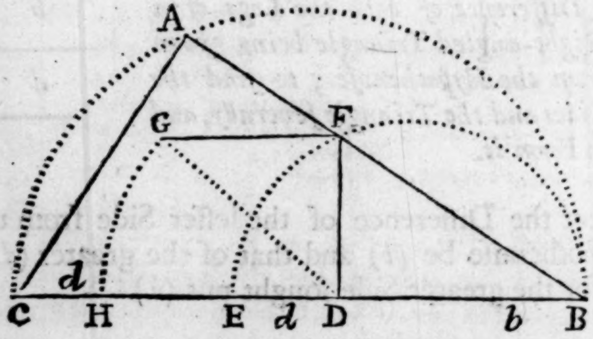
Then will,

	1	$a + d =$ Hypothenuſe. And,
$1 - b$	2	$a + d - b =$ to the leſſer Side.
47. *e.* 1.	3	$aa + 2ad + dd := 2aa + 2ad - 2ab - 2bd + dd + bb.$
	4	$aa - 2ab - 2bd + bb = 0,$ by Compariſon and Tranſpoſition of the laſt Step.

Tranſp.	5	$aa-2ab=2bd-bb$, which is a Quadratick Equation of the ſecond Form.
Compl. □	6	$aa-2ab+bb=2bd$.
w.	7	$a-b=\sqrt{:2bd}$.
		$a=\sqrt{:2bd}+b$.

The Geometrical Conſtruction.

Find a mean Proportional between *d* and *b*, which let be D F; to which, place at Right-angles F G = to D F, draw G D, and cut off H D = G D. Then will B H be the greater Side ſought. And this being produced to C (ſo that C H = E D) will give C D (= A C) the leſſer Side of the Triangle required. Draw a Semicircle on C B, and apply A B = H B. Then draw A C, and the Triangle is found, which is A C B.



ing yͤ notations, thus, divide yͤ 4th equation by e, & twill be

4 ÷ e | 18 | $2aa + 2a + e + {}^{2}a - {}^{2}e = 38$

18 ÷ 2 | 19 | $aa + a + \frac{1}{2}e + a - e = 19$

Trans: | 20 | $aa + 2a - \frac{e}{2} = 19$

Trans: | 21 | $aa + 2a = 19\frac{1}{4}$ Which is a Quadratick of the first form.

22 | $\sqrt{20\frac{1}{4}} = 4\frac{1}{2} = a + 1$

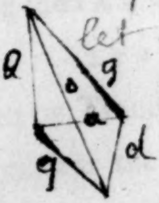
22 − 1 | 23 | $a = 3\frac{1}{2}$

a + e | 24 | $a + e = 4$

a − e | 25 | $a - e = 3$.

A Problem

Having the content & 4 sides of an irregular Trapezium that has no right angles, 'tis requir'd to find yͤ diagonals



let yͤ lesser diagonal = a
yͤ greater diagonal = o
the content = δ
the 4 sides = d, q; b, g

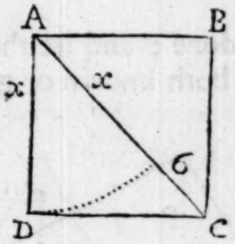
Then will $\frac{qq + bb - aa}{2b}$ (vid: pag: 45) = the ~~point~~ distance of yͤ point where a perpendicular let fall from yͤ vertex of yͤ triangle eba will touch b. Then will

$\sqrt{ee - \frac{eeee - 2bbee + 2aaee - bbbb + 2bbaa - aaaa}{4b}}$

$\times \frac{\cancel{eee} + bb - aa}{4b} +$.

PROBLEM II.

Having in the Square A B C D, *the Difference between the Sides and Diagonal* = 6, *or* a, *to find the Side of the Square.*



Let the Side ſought be called x, and $6 = a$.

Then $x + a = AC$ the Diagonal.

But (by 47 e. 1 *Eucl.*) $ACq; = 2ADq;$ or to $2xx$.

That is $xx + 2xa + aa = 2xx$.

Expunge then xx on both Sides, and it will be

$$2xa + aa = xx.$$

And then by Tranſpoſition, $xx - 2ax = aa$.

Compleat the Square, and it will be

$$xx - 2ax + aa = 2aa.$$

Wherefore $x - a = \sqrt{:2aa}$,

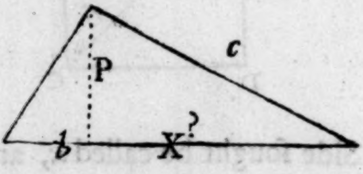
And conſequently $x = \sqrt{:2aa} + a = 14.48$.

PROB.

PROBLEM III.

Given one Segment of the Baſe of a Right-angled Triangle, as alſo the Side of the Triangle Adjacent to the other Segment of the Baſe; 'tis required to find the reſt, and to form the Triangle.

Suppoſe it done; and let the Segment b, and the Side c, be both known or given. Let x, the other Segment of the Baſe, be ſought; which is all that is neceſſary to ſolve the *Problem.*



Here therefore, ſince P is ſuppoſed to be a true Perpendicular.

1 $cc - xx = pp$, 47 e. 1 *Euclid.*

2 And becauſe the Angle at the Top is a Right one, therefore $pp = bx$, which gives another way of expreſſing pp. So that,

3 $cc - xx = bx$, and conſequently by Tranſpoſition.

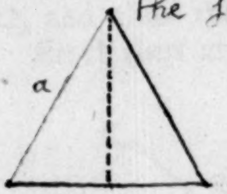
4 $cc = xx + bx$, which is an Adfected Quadratick of the firſt Form. Wherefore,

5 $cc + \frac{bb}{4} = xx + bx + \frac{bb}{4}$ by compleating the Square. And,

A Problem.

Having yͤ content of an equilateral triangle how to find the sides & form yͤ triangle is requird.

Let the content $= b$
the side sought $= a$



Then will $\sqrt[2]{aa - \frac{aa}{4}} \times \frac{a}{2} = b$

~~[illegible]~~ $1 \div \frac{a}{2}$	2	$\sqrt[2]{aa - \frac{aa}{4}} = \frac{2b}{a}$
	3	$aa - \frac{aa}{4} = \frac{4bb}{aa}$
$3 \times aa$	4	$\overset{aaa}{a}_{\wedge} - \frac{1\,\overset{aaaa}{}}{4_{\wedge}} = 4bb$ that is $\frac{3aaaa}{4} = 4bb$
$4 \times 1\frac{1}{3}$	5	$aaaa = 5bb + \frac{bb}{3}$.
$\sqrt[4]{:}5$	6	$a = \sqrt[4]{:5bb + \frac{bb}{3}}$

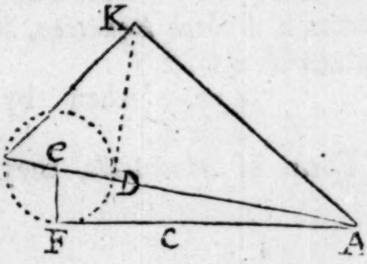
6 $\sqrt{cc+\frac{bb}{4}}=x+\frac{b}{2}$, by Evolution.

Laſtly,

7 $\sqrt{cc+\frac{bb}{4}}-\frac{b}{2}=x$.

Geometrical Conſtruction.

Join together at Right Angles $eF=\frac{1}{2}b$ and F A $=$ C. Then with the Radius e F deſcribe the Circle B F D, and 'thro' the Centre e draw the Line A D B. Erect then at D the Perpendicular D K, which Limit, by deſcribing a Semicircle on B A: That Semicircle ſhall cut the Perpendicular in the Point K, the Vertex of the Triangle required, whence draw the two Legs B K and K A. So is B K A the Triangle ſought.



Of Cubick and Biquadratick Equations.

MR. *Harriot* ſhews the Original of a *Cubick Equation* to be derived either from three Lateral or Simple Equations, reduced firſt to the Form of *Binomials*, and then Multiplied continually into each other; or elſe from one Quadratick Multiplied by a Lateral.

Whence he deduces that all *Cubick Equations* have Real or Imaginary, 3 Roots; or as many as are the Dimenſions of its higheſt Power.

Thus to form a *Cubick Equation*, let its three Roots or Values be $\left.\begin{array}{l} a = 2 \\ a = 3 \\ a = 4 \end{array}\right\}$ then by reducing them to his Form of *Binomials*, they will ſtand thus,

$$\begin{array}{l} a - 2 = 0 \\ a - 3 = 0 \\ a - 4 = 0 \end{array}$$

And theſe 3 *Binomials* Multiplied continually into one another, do produce this Equation $aaa - 9aa + 26a - 24 = 0$, or $a^3 - 6aa + 26a = 24$. Which *Cubick Equation* would have been produced alſo by Multiplying the Quadratick $aa - 5a + 6 = 0$, by $a - 4 = 0$.

In like manner he ſhews the Derivation of a *Biquadratick Equation*, to be either from four Simple

Of

Cubick & Biquadratick equations.

1 From these 3 roots $a=2$ $a=3$ & $a=-5$ being reduc'd to ye form of Binomials thus $a-2=0$ $a-3=0$ $a+5=0$ wil arise this cubick equation. $aaa+6a-25a+30$ or $aaa-19a+30$ $=0$: where ye 2d term is wanting.

This last equation multiply'd by $a-4=0$ will be a Biquadratick, viz. $aaaa-4aaa-19aa+106a-120=0$

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Simple or Lateral Equations, reduced as above, to the Form of *Binomials*, and continually Multiplied into one another: Or elſe from a *Cubick* into a *Lateral*, one Quadratick into another, or a Quadratick Multiplied by 2 Laterals. Wherefore he ſaith, every Biquadratick will have Real or Imaginary, four Roots, agreeable to the Dimenſions of its higheſt Power.

Thus if the former Cubick $a^3 - 9aa + 26a - 24 = 0$, be Multiplied by $a + 5 = 0$, there will ariſe this *Biquadratick Equation*, $a^4 - 4a^3 - 19aa + 106a - 120 = 0$; that is, $a^4 - 4a^3 - 19aa + 106a = 120$.

From which Original of theſe *Cubick* and *Biquadratick Equations*, 'tis plain, That aſſoon as you can diſcover the Value of any one Root, you may depreſs the Equation a Dimenſion lower, by dividing it by ſuch Root reduced to the Form of a Binomial, as above. Thus, If you find that one Root, or one a is $= 2$, then divide the laſt Equation by $a - 2$, and it will bring it down to a *Cubick*; and that *Cubick* being again divided by $a - 3$, $a - 4$, or $a + 5$, will be depreſſed into a *Quadratick*, &c. And this is ſometimes of good Uſe to diſſolve *Compound Equations* into their Components, as hath been ſhewn by *Des Cartes*, *Hudd*, and others.

From this Method of Compoſition of theſe Equations, 'tis alſo apparent, of what Members each of the Coefficients are made up. For,

I. *The Coefficient of the ſecond Term, is always the Aggregate of all the Roots under contrary Signs.* Thus, In the *Cubick Equation* above mentioned, the Coefficient 9, is the Sum of 2, 3, and 4, with the Negative Sign. And 4 the Coefficient of the ſecond Term in the ***Biquadratick Equation*** above mentioned, is the Aggregate of 2, 3, 4 and — 5; that is, 4 with a Negative Sign. Wherefore it follows, That if all the Negative Roots, ſecluding their Signs, be equal to all the Affirmatives; (tho' not each to each reſpectively) then will the Second Term quite vaniſh out the Equation, and be *wanting*, as 'tis call'd; becauſe the Negatives and Affirmatives do mutually deſtroy each other: And *vice versâ*, whenever the ſecond Term is *wanting* in one of theſe Equations, the Roots are thus equal, and have contrary Signs.

II. The Coefficient of the Third Term is the Aggregate of all the Rectangles made by the Multiplication of every pair of the Roots, as often as they can be taken; which in a *Cubick* is 3, in a *Biquadratick* 6, in an Equation of the fifth Power 10, *&c.* according to the Order of Triangular Numbers.

Thus in the Third Term 26 *a* of the *Cubick Equation* before mentioned; 26 the Coefficient is the Aggregate of 6, 8 and 12, the 3 Rectangles of the Roots 2, 3 and 4.

And here if all the Negative Rectangles (ſecluding their Signs) are equal to all the Affirmative ones, they will deſtroy one another, and ſo the Third Term will vaniſh or be wanting.

III. The Coefficient of the fourth Term, is the Aggregate of all the Solids made by the continual Multiplication of all the Ternary's, or every three of ſuch Roots ſo Signed, *&c.* and ſo on *ad Infinitum.*

IV

IV. As in *Quadraticks*, the Abſolute Number given, is always the Rectangle of the 2 Roots, or Values of *a*. So in *Cubicks* 'tis always the *Solid* made by continual Multiplication of all the three Roots one into another; and in *Biquadraticks*, of all the four Roots, &c.

The Reſolution of Cubick and Biquadratick Equations.

AS to the Reſolution of theſe kind of Equations, I ſhall in this ſhort Treatiſe make no mention of *Kerſey*'s way of finding all the juſt Diviſors to the laſt Term, or Abſolute Number given; (*See* Kerſey's *Algebra, Vol.* 1. *Book* 2. *Chap.* 10. *Sect.* 9.) Nor of the Common Rules of *Cardan* for *Cubicks*: Where the ſecond Term muſt firſt be taken away; becauſe at the beſt they are not perfect; and alſo becauſe they are very tedious and troubleſome.

I would rather adviſe the Learner to make Uſe in ordinary Caſes of the general Method of *Stevinus* mentioned by *Kerſey* in the ſame Chapter: For tho' that be but a Tentative Way, and comes to the Truth only by frequent Trials; yet when 'tis made familiar by Practice, and Experience hath taught him how to judge of the Limits of Equations, it will expeditiouſly enough diſcover one true Root (if ſuch there be) in almoſt any kind of Equation; whether having all its *Parodick*

Degrees,

Degrees, or Terms, or not. And when one true Root is found, you may (as is ſhewn above) depreſs the given Equation by it (if it be a Regular one) one Degree lower, and by that means eaſily diſcover the other Roots.

Of this Method take a few Examples.

EXAMPLE I.

Suppoſe the former Cubick Equation *given*, $a^3 - 9 aa + 26 a = 24$.

Firſt, I will imagine $a = 1$, and working according to the Equation, I find that $27 - 9 (= 18) = 24$. Wherefore I conclude *a* is greater than I try again, and ſuppoſe $a = 2$. Then will $8 + 52 - 36$ (*i. e.* $60 - 36$) $= 24$. Which Anſwers my deſire, and gives me one real Value of *a* the Root in this *Cubick Equation*: After which I may either divide the given Equation by $a - 2$, which will bring it down to a *Quadratick*; or I might have proceeded further in the ſame Method, and have found alſo, that 3 and 4 would have been the other 2 real Roots.

EXAMPLE II.

Suppoſe $aaa - 22 aa + 157 a = 360$?

I make tryals with 1, 2, 3 and 4, and find them all too little; wherefore I imagine $a = 5$. Therefore according to the Equation, $aaa (= 125) + 157 a (= 785) - 22 aa (= 550) = 360$; which I find it is exactly: And thence I conclude, that 5 is

Of ye Infinite Converging Series.

Let $\begin{cases} A = \text{ye absolute Number in any Equation.} \\ n = \text{ye exponent of ye highest power.} \\ x = \text{root or quantity sought.} \\ n = \text{an unknown Number.} \\ N = \text{any known Number taken at pleasure.} \end{cases}$

& 1, p, q, r, s, &c = ye respective Coefficients of ye given Equation.

Then will $N+n=x$ & a General Equation, representing any one whatsoever, may be thus express'd $\mp 1 \times \overline{N+n}|^{n} \mp p \times \overline{N+n}|^{n-1} \mp q \times \overline{N+n}|^{n-2} \mp r \times \overline{N+n}|^{n-3} \mp s \times \overline{N+n}|^{n-4} \mp$ &c. $= A$.

But to bring this into a Series, it is to be first prov'd, yt every power rais'd from a Binomial, (without regarding ye Coefficients) is compos'd of 2 ranks of powers; one increasing from n^{n-n}, or 1 to n^n, & ye other decreasing from N^n to N^{n-n} or 1, & each member is multiply'd into it's corresponding member respectively, as may appear thus:

$$\overline{N+n}|^{3} = \begin{Bmatrix} NNN \\ 3.NNn \\ 3.Nnn \\ nnn \end{Bmatrix} = \left\{\begin{array}{l} N^{n} \times n^{n-n} \text{ or } 1 \\ \left.\begin{array}{l} N^{n-1} \times n^{n-2} \\ N^{n-2} \times n^{n-1} \end{array}\right\} \text{Thrice.} \\ N^{n-n} \times n^{n} \end{array}\right.$$

(See ye table of Powers inter pag. 128am & 129am.)

& so on ad Infinitum. Hence 2 Corollaries.

Corollary 1st.

That ye Coefficient of ye 2d term, in any power rais'd from a Binomial, alwaies = n ye exponent of ye highest power.

Corolis one true Root of that Equation. Dividing then the Equation $aaa - 22aa + 157a - 360 = 0$ by $a - 5$. I reduce it to this Quadratick $aa - 17a + 72 = 0$. That is, $aa - 17a = 72$, whoſe two Roots are 8 and 9: Wherefore I conclude 5, 8 and 9, to be the three Roots of the Etion given.

If this irregular Equation were propoſed, (where alſo the Abſolute Number is a Fraction) $xxxx + 50x = 184638.6801$. I can diſcover at firſt Sight almoſt, that x muſt be at leaſt equal to 10, and trying with 10, I find it too little; but trying with 100, I find that much too great: Proceeding then again, I find 30 too much; I try 20, and I find that ſomething too ſmall; but 21 I find too big: Wherefore I know x muſt be = to 20, with ſome Decimal Fraction annexed. And at laſt I diſcover 20.7 to be the very Root ſought: For Multiplying *that* according to the Equation, it will produce $xxxx + 50x = 184638.6801$.

But for an Univerſal Method of Extracting the Roots out of all manner of Equations, whether *Pure* or *Adfected*, in Numbers, there muſt be recourſe had to that of an *Infinite* or *Converging Series*. Which, I believe was firſt hinted by the Incomparable Mr. *Iſaac Newton*, and afterwards purſued very fully by Mr. *Ralphſon*; and from him by D. *Lagney*; then the Sagacious Mr. *Halley* took the Matter into Conſideration (in *Philoſoph. Tranſact.* N. 210. *A. D.* 1694.) Demonſtrating the Reaſon of Dr. *Lagney*'s Rules; and carrying the Thing much farther, by an Univerſal Method of his own. Which becauſe 'tis there largely delivered, Illuſtrated with Examples, and Explained by proper Notes and Obſervations: And becauſe alſo the Ingenious Mr. *Wells* of *Oxford*, hath given a very good Account of it in his *Element. Arith. Numer. & Specioſ.* I ſhall not here ſtay to explain, but muſt refer the *Learned Reader* thither. And this I the rather chuſe to do, becauſe Mr. *Ward* alſo in his *Compendium of Algebra*, hath largely inſiſted on this Subject, in our own Language; hath very much improven on Mr. *Ralphſon*'s and Capt. *Halley*'s Foundation, and given a variety of Theorems and Examples; with very uſeful *Contractions* in the ſeveral Methods of Operation.

I ſhall therefore next proceed to give you the Geometrical Conſtruction of theſe kinds of Equations, by which *All their real Roots* will be moſt eaſily and readily found.

Conſtruction of Cubick and Biquadratick Equations.

SInce the Conſtruction of theſe kinds of Equations is done by the help of the *Parabola*: It will be neceſſary firſt to Explain what is meant by that Word, and to ſhew you thoſe Properties of it, which are made uſe of in theſe Conſtructions.

If then a Cone, as *a b c* be cut thro' its Axis, the Section will be a Triangle, as *a b c*, and if in the Plane of that Triangle you draw a Line, or A *x* X Parallel to either ſide of the Cone; as ſuppoſe here to *a c*: And then in the Plane of the Circular Baſe of the Cone, erect X N Perpendicular

Corollary 2d.

That ye Root or side of ye $^{n^n}$ unknown quantity is alwaies multiply'd ^ into ye 2d term of ye known. now from ye latter tis evident we are in this Case to make use of but ye 2 first members of ye power of such Binomial; & by ye first we may express ye Co-efficient of ye 2d term by ye exponent of ye power. therefore ye former General Equation will now stand thus;

$$\mp 1\times N^{n} \pm {}_{n}N^{n-1}n \mp\pm p\times N^{n-1} \pm {}_{n-1}N^{n-2}n \mp\pm q\times N^{n-2} \pm {}_{n-2}N^{n-3}n \mp\pm r\times N^{n-3} \pm {}_{n-3}N^{n-4}n \mp\pm s\times N^{n-4} \pm {}_{n-4}N^{n-5}n \mp\pm \&c. = A.$$

Now to find ye value of n ye unknown quantity, those members into wch it is multiply'd, will be ye divisor wth ye same signs, & ye others be ye Dividend, but wth contrary signs, as being to be transposed to ye other side of ye Equation: therefore 1st,

Theorem 1st.

$$n = \frac{A \mp 1\times N^{n} \mp p\times N^{n-3} \mp q\times N^{n-2} \mp r\times N^{n-3} \mp s\times N^{n-4}}{1\times {}_{n}N^{n-1} \pm p\times {}_{n-1}N^{n-2} \pm q\times {}_{n-2}N^{n-3} \pm r\times {}_{n-3}N^{n-4} \pm s\times {}_{n-4}N^{n-5} \;\&c.}$$

Which Theorem exhibits all possible particular ones, for

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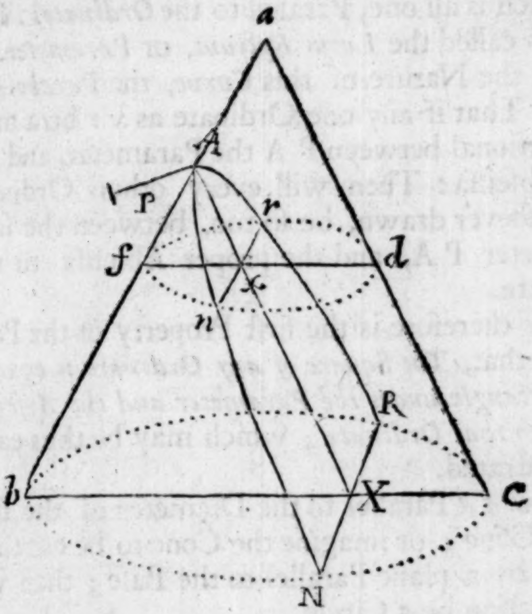


cular to the Diameter. I ſay, If you imagine the Cone to be now cut by another Place, according to the two Right-lines A X, X N, the Section ariſing from thence, N *n* A *r* R N is called a *Parabola.*

The Point A is called the *Vertex* of the Section, or the *Vertex* of the *Parabola.*

Any Right-line, as *r x*, or R X; *n x* or N X; applied at Right Angles to A X (which is called the Axis of the *Parabola*) is called an *Ordinate*: And the Part of the Axis intercepted between the Vertex A and the Ordinate, is called the *Abſciſſa*: By ſome, the *Intercepted Axe.*

And if you imagine a Right-line found, as P A, which ſhall be a third Proportional to the *Abſciſſa*, A *x* and *Ordinate*, *r x*; and which ſhall be placed at the Vertex A, Perpendicular to the Axis, or which is all one, Parallel to the *Ordinates*: That Line is called the *Latus Rectum*, or *Parameter*.

And the Nature of this *Curve*, the *Parabola*, is ſuch : That if any one Ordinate as $x\,r$ be a mean Proportional between P A the Parameter, and A x the Abſciſſa: Then will every other Ordinate, whereſoever drawn, be ſo too, between the ſame Parameter P A, and the proper Abſciſſa to that Ordinate.

This therefore is the firſt Property of the Parabola ; that, *The Square of any Ordinate is equal to the Rectangle under the Parameter and the Abſciſſa, proper to that Ordinate* ; which may be thus eaſily demonſtrated.

Draw $f\,d$ Parallel to the Diameter of the Baſe of the Cone ; or imagine the Cone to be cut there again, by a plane Parallel to the Baſe ; then will that Section be a Circle.

1 'Tis plain $x\,d$ will be $=$ X c, as being oppoſite Sides of a Parallelogram.

2 The Square of X R $=$ Rectangle b X c, and the Square of $x\,r$ $=$ Rectangle $f\,x\,d$, from the Nature of the Circle.

3 A X : X b : : A x : $x\,f$, by Similar Triangles.

4 Wherefore if X b and $x\,f$ be Multiplied by one and the ſame length, the Proportion will continue, and it will ſtand thus, A X : X b × X c : : A x : $x\,f$ × $x\,d$.

5 That is in other Words, A X : X R q; : : A x : $x\,r$ q; by Step 2 ; and therefore alternately, A X : A x : : X R q; : $x\,r$ q;. Which by the by, ſhews you alſo another general

for extracting of Roots according to yᵉ 1st sort of Mr Ralphson's, agreeing exactly with them; alwaies remembring, that yᵉ signs in yᵉ Dividend must be contrary to those in yᵉ Equation, & those in yᵉ Divisor yᵉ same respectively.

But $N + n = x$: therefore 2dly,

Theorem 2d.

$$x = \frac{A \pm 1x_{n-1}N^{n} \pm px_{n-2}N^{n-1} \pm qx_{n-3}N^{n-2} \pm rx_{n-4}N^{n-3} \pm sx_{n-4}N^{n-4}, \&c.}{1x_{n}N^{n-1} \pm px_{n-1}N^{n-2} \pm qx_{n-2}N^{n-3} \pm rx_{n-3}N^{n-4} \pm sx_{n-4}N^{n-4}, \&c.}$$

In this case yᵉ signs both in yᵉ Dividend & Divisor, will be yᵉ same as in yᵉ given Equation respectively.

N.B. If any term be wanting in yᵉ Equation, yᵉ same must be omitted in either Theorem respectively.

Now from either of these two Generals to deduce any particular Theorem, for finding yᵉ Root of any given Equation; consider, that $N^{n-n} = 1$, or $\frac{N^n}{} = 1$, that unity will neither multiply N^n nor divide; as also that ${}_{n-n}N^n = 0$ or ${}_nN^n - {}_nN^n = 0$ & any quantity multiply'd into $0 = 0$, & when either case happens, (wᶜʰ alwaies will, except where yᵉ last Term is wanting) yᵉ Theorem is determin'd.

Theore-

Suppose any Equation, as	Therefore, By ye 1st General Theore.	By ye 2d General Theor each
$x^2 = A.$	$\frac{A - N^2}{2N}$	$\frac{A + N^2}{2N}$
$x^3 = A.$	$\frac{A - N^3}{3N^2}$	$\frac{A + 2N^3}{3N^2}$
$x^4 = A.$	$\frac{A - N^4}{4N^3}$	$\frac{A + 3N^4}{4N^3}$
$x^2 \pm px = A.$	$\frac{A - N^2 \mp pN}{2N \pm p}$	$\frac{A + N^2}{2N \pm p}$
$-x^2 \pm px = A.$	$\frac{A + N^2 \mp pN}{-2N \pm p}$	$\frac{A - N^2}{-2N \pm p}$
$x^4 \pm px^3 = A.$	$\frac{A - N^4 \mp pN^3}{4N^3 \pm 3pN^2}$	$\frac{A + 3N^4 \pm 2pN^3}{4N^3 \pm 3pN^2}$
$x^3 \pm px^2 \pm qx = A.$	$\frac{A - N^3 \mp pN^2 \mp qN}{3N^2 \pm 2pN \pm q}$	$\frac{A + 2N^3 \pm pN^2}{3N^2 \pm 2pN \pm q}$
$x^4 \pm px^3 \pm qx^2 \pm rx = A.$	$\frac{A - N^4 \mp pN^3 \mp qN^2 \mp rN}{4N^3 \pm 3pN^2 \pm 2qN \pm r}$	$\frac{A + 3N^4 \pm 2pN^3 \pm qN^2}{4N^3 \pm 3pN^2 \pm 2qN \pm r}$
$x^5 \pm px^4 \pm qx^3 \pm rx^2 \pm sx = A.$	$\frac{A - N^5 \mp pN^4 \mp qN^3 \mp rN^2 \mp sN}{5N^4 \pm 4pN^3 \pm 3qN^2 \pm 2rN \pm s}$	$\frac{A + 4N^5 \pm 3pN^4 \pm 2qN^3 \pm rN^2}{5N^4 \pm 4pN^3 \pm 3qN^2 \pm 2rN + s}$
$x^4 \pm rx = A$	$\frac{A - N^4 \mp rN}{4N^3 \pm r}$	$\frac{A + 3N^4}{4N^3 \pm r}$
&c.	&c.	&c.

Then we shall find n =

x

general Property of the ***Parabola***; that, *The Squares of the Ordinates are to one another as the Abſciſſe.* Vid. Mydorg. Conicorum Lib. 1. Theor. 7.

6 If four Quantities be proportional, they may be expreſſed Fraction-wiſe thus, $\frac{XRq;}{AX} = \frac{xrq;}{Ax}$. And then either of thoſe Quantities will expreſs the *Parameter* or *Latus Rectum*: Let each therefore be called P. Wherefore ſince $\frac{XRq;}{AX} = P : P \times AX$ will be equal to $XRq;$: and $P \times$ by $Ax = xrq;$

7 Wherefore $P : XR :: XR : AX$, and $P : xr :: xr : Ax$.

That is in Words, *The Ordinate is a mean Proportional between the Parameter and the Abſciſſa.*

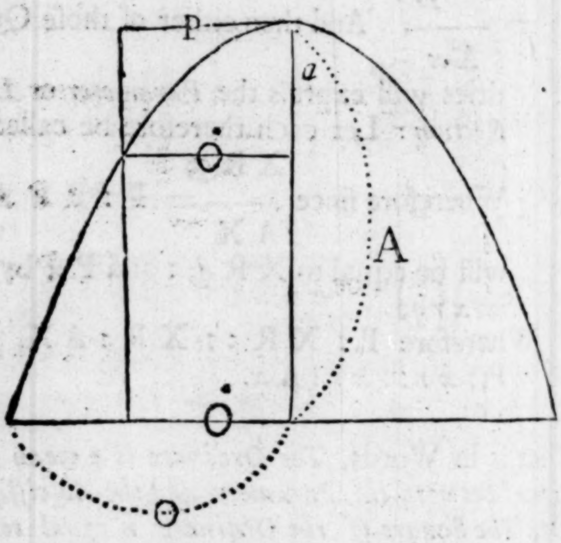
Or, *The Square of the Ordinate is equal to the Rectangle under the Parameter and Abſciſſa.* Which is the firſt Property of the *Parabola* below to be made uſe of.

The ſecond Property of the *Parabola*, here to be obſerved, is this; that, *The Parameter is to the Sum of any two Ordinates, as their Difference is to the Difference of the Abſciſſa.*

Which Property, now commonly called by the Name of *Baker*'s Property, was unknown to the Ancients, and was diſcovered by Mr. *Strode* of *Maperton*, who communicated it to Mr. *Baker*, as he Ingenuouſly owns. Tho' Mr. *Baker* applied i with great Advantage to the Conſtruction of *Cu bick* and *Biquadratick Equations*, having all thei Terms, as we ſhall ſhew below.

This Property is thas demonſtrated very briefly.

Let p be the Parameter, O and $\dot{O}$ two *Ordinates*, and A and a their proper *Abſciſſa*.



I ſay, That $P : O + \dot{O} :: O - \dot{O} : A - a$.

For,

1 $PA = OO$ and $Pa = \dot{O}\dot{O}$, by the firſt Property of this *Curve*.

2 Wherefore $PA - Pa = OO - \dot{O}\dot{O}$, and then by only,

3 Reſolving that Equation into Proportionals, it will ſtand thus.

$P : O + \dot{O} :: O - \dot{O} : A - a$. Q.E.D.

Becauſe

Thus having ye particular Theorem, ye Application in either case is as follows:

Let N = any Number taken at pleasure, as before.

T = Theorem in wch N must alwaies be of it's last value found.

Then ye process will be of ye

1st Gen. Theorem.	2d General Theorem.
N ye 1st ± T = N ye 2d, then	T = N ye 2d, Then
N ye 2d ± T = N ye 3d, then	T = N ye 3d, then
N ye 3d ± T = N ye 4th, then	T = N ye 4th, Then
N ye 4th ± T = N ye 5th &c.	T = N ye 5th &c.

Some of wch values of N will terminate in ye true Root sought, unless it be a Surd; then ye value of N will proceed into an infinite Series, wch may be prosecuted still nearer & nearer ye truth, ye number of places increasing thus, 1, 2, 4, 8, 16, 32, &c places.

Tis observable yt ye 1st Theorem converges by finding a Number to be added to, or Substracted from ye last value of N, as it shall be affected wth + or — till $N = x$: but ye 2d converges by N it self, till it $= x$ sought.

Exam-

Examples.

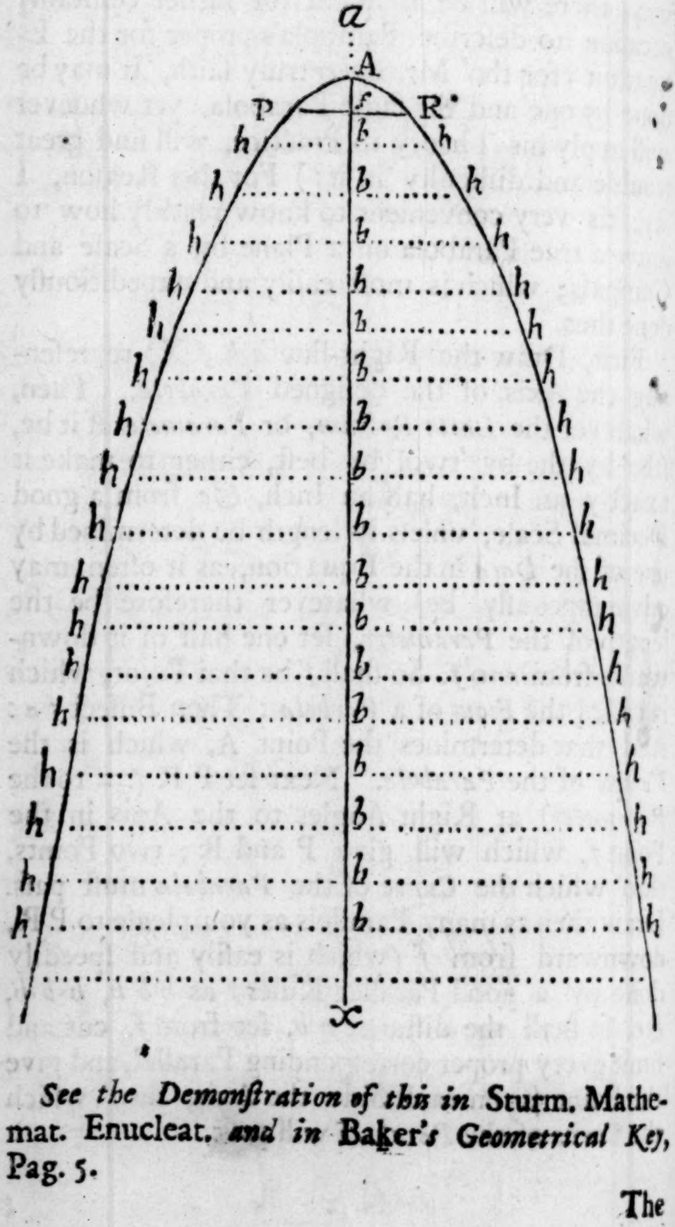
I. $x^2 = 3$. Let x be sought by Theorem ye 1st. Then will $\frac{A - N^2}{2N} = T$. Let $N = 1$.
Then $N + T (= 1) = 2 = N\ 2^d$.
Then $N\ 2^d - T (= -,2) = 1,8 = N\ 3^d$.
Then $N\ 3^d - T (= -,06) = 1,74 = N\ 4^{th}$.
Then $N\ 4^{th} - T (= -,007) = 1,733 = N\ 5^{th}$
Then $N\ 5^{th} - T = -,0009) = 1,7321 = N\ 6^{th}$
$= x$ nearly. If we seek x by Theorem ye 2d Then will $\frac{A + N^2}{2N} = T$. Let $N = 1$,
Then $T = 2 = N\ 2^d$. | Then $T = 1,732 = N\ 5^{th}$.
Then $T = 1,7 = N\ 3^d$ | Then $T = 1,7320 = N\ 6^{th} =$
Then $T = 1,73 = N\ 4^{th}$ | x nearly.

The difference between ye 2 Theorems, in ye last value of N is only in ye last figure, wch yet would have been set right ye next operation.

II. $x^3 = 2,197$. Let x be sought by Theorem ye 1st. Then $\frac{A - N^3}{3N^2} = T$. Let $N = 1,5$.
Then $N - T (= ,17) = 1,33 = N\ 2^d$
Then $N\ 2^d - T (= 0,0293) = 1,3007 = N\ 3^d$
Then $N\ 3^d - T (= ,0007) = 1,3 = x$ sought.
Let x be sought by Theorem 2d. Then $\frac{A + 2N^3}{3N^2} = T$. Let $N = 1,5$
Then $T = 1,32 = N\ 2^d$ | Then $T = 1,3 = x = N\ 4^{th}$.
Then $T = 1,3003 = N\ 3^d$ |
So the latter Theorem gives ye Root at one o-

Becauſe in the Conſtructing of Equations this way, there will be frequent (or rather conſtant) occaſion to deſcribe Parabola's proper for the Equation (for tho' Mr. *Baker* truly ſaith, It may be done by one and the ſame Parabola, yet whoever will apply his Theory to Practice, will find great trouble and difficulty in it:) For this Reaſon, I ſay, 'tis very convenient to know readily how to draw a true Parabola on a Plane by a Scale and Compaſs; which is moſt eaſily and expeditiouſly done thus.

Firſt, Draw the Right-line *a* A *f* X, repreſenting the Axis of the deſigned *Parabola.* Then, whatever the *Latus Rectnm*, or *Parameter* if it be, (tho' by the by 'twill be beſt, either to make it exactly an Inch, half an Inch, *&c* from a good Decimal Scale, unleſs its length be determined by one of the *Data* in the Equation, as it often may advantageouſly be) whatever therefore be the length of the *Parameter*, ſet one half of it downwards from *a* to *f*. So ſhall *f* be that Point, which is called the *Focus* of a *Parbola* : Then Biſſect *f a* : And that determines the Point A, which is the *Vertex* of the *Parabola.* Next ſet P R (= to the *Parameter*) at Right Angles to the Axis in the Point *f*, which will give P and R; two Points, thro' which the *Curve* of the *Parabola* muſt paſs. Draw then as many Parallels as you pleaſe to P R, downward from *f* (which is eaſily and ſpeedily done by a good Parallel Ruler) as *b b b*, *b b b*, &c. ſo ſhall the diſtance *a b*, ſet from *f*, cut and limit every proper correſponding Parallel, and give the Point *b*, on each Side the Axis, thro' which the *Curve* of the *Parabola* will paſs.



See the Demonſtration of this in Sturm. Mathemat. Enucleat. *and in* Baker's *Geometrical Key*, Pag. 5.

operation more than y^e former:
for by that $N^o\,3^d = x$, but by this $N^o\,4^{th} = x$.

III. $x^3 - 9x^2 + 26x = 24$. (which is y^e 3^d Example
given page 8^{th} of this book.) Let x be sought
by Theorem y^e 1^{st}. Then $\frac{A - N^3 + 9N^2 - 26N}{3N^2 - 18N + 26} = T$.
for $\frac{A - N^3 - pN^2 - qN}{3N^2 - 2pN + q} = T$. Let $N = 1$. Then

$N + T\,(= +,5) = 1,5 = N^o\,2^d$. Then
$N^o\,2^d + T\,(= ,32) = 1,82 = N^o\,3^d$. Then
$N^o\,3^d + T\,(= +0,1394) = 19594 = N^o\,4^{th}$. Then
$N^o\,4^{th} + T\,(= ,03834123) = 1,99774123 = N^o\,5^{th}$. ~~Then~~
$= x$ nearly. To pursue y^e Search farther
would be troublesome, because of y^e
Number of places in y^e last Value of N:
but y^e absolute Number 24, being an
Integer, tis an Indication y^t x) is so too
take therefore y^e next whole Number
to $1,99774123\,(= N^o\,5^{th})$, w^{ch} is 2 & it $=$
x sought.

The firſt that attempted the general Conſtruction of theſe Equations, was the Famous *Des Cartes*; who in the third Book of his excellent Geometry, gives a Method by means of a Parabola and a Circle, to conſtruct and find the real Roots of all Equations not exceeding four Dimenſions. Which Method was however not perfect, becauſe it would conſtruct only ſuch Cubick and Biquadratick Equations as had their ſecond Term firſt taken away.

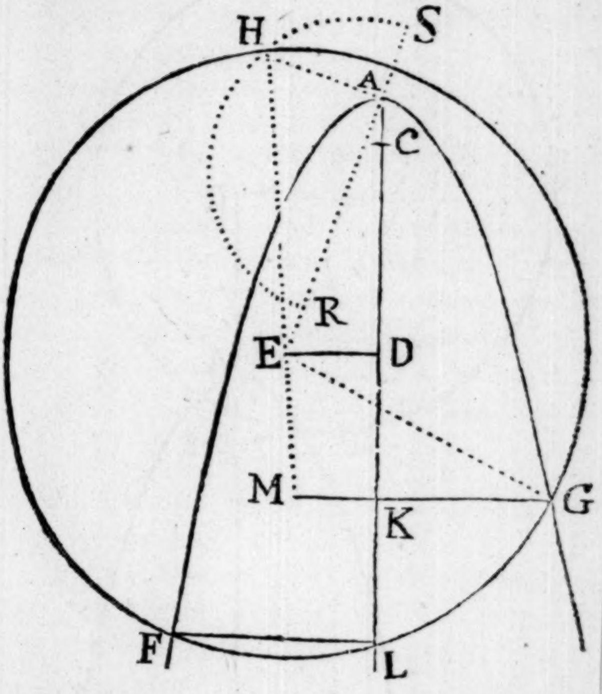
However, becauſe this was it which gave Riſe to *Baker*'s excellent Rule, and to whatever Improvements have been ſince made in it; and becauſe here is laid the firſt Foundation, Reaſon, and Demonſtration of the whole Matter; I ſhall begin with a ſhort account of it.

When the ſecond Term is taken away, or wanting, he reduces all *Cubick Eqnations* to this Form $Z^3 * apz : aaq = 0$: And all *Biquadraticks* to this, $Z^4 * apzz : aaqz : aaar = 0$. Where a repreſents the *Latus Rectum* or *Parameter* of any given *Parabola*; and is ſuppoſed $= 1$. That ſo its Powers may produce no trouble in the Operation. By which means the former Equations will be in this Form, $Z^3 * pz : q = 0$, and $Z^4 * pzz : qz : r = 0$. Where z is the unknown Root ſought; p the known Part of the *Third Term*, or a known Number Multiplied into the Square of z: q the known Part of the *Fourth Term*, or another known Number Multiplied into z, and r (if it be a *Biquadratick*) is the *Fifth Term*, or Abſolute Number given. But if the ſecond Term had not been wanting, that would have been p, the third q, the fourth r, &c. and in the *Cubick*, q is the abſolute Number.

Let then any Parabola, as F A G, be ſuppoſed to be deſcribed, whoſe Axis is A D L, and its Parameter $a = 1$.

Firſt, Take A C equal to half a; ſo that the Point C will always be within the Parabola. Next in the Axis, (downwards from C if p have a Negative Sign, but upwards in the Axis produced when p hath a Poſitive Sign) take $CD = \frac{1}{2}p$. Then from the Point D thus found, (or from C if the known Part of the third Term q be alſo wanting in the Equation) erect a Perpendicular to the Axis, as D E, and make it equal to half q: Which Perpendicular D E, muſt be on the Right-hand of the Axis, if q have a Negative Sign: But towards the Left if it be $+q$. After which deſcribing a Circle on the Center E, with the Radius E A, it will (if the Equation were only a *Cubick* one) cut the Parabola in as many Points as the Equation hath true Roots: And the Affirmative ones will be Perpendiculars or Ordinates, let fall from the Curve to the Axis on the Right hand, and the Negative ones ſuch, ſo let fall to the Axis on the Left hand.

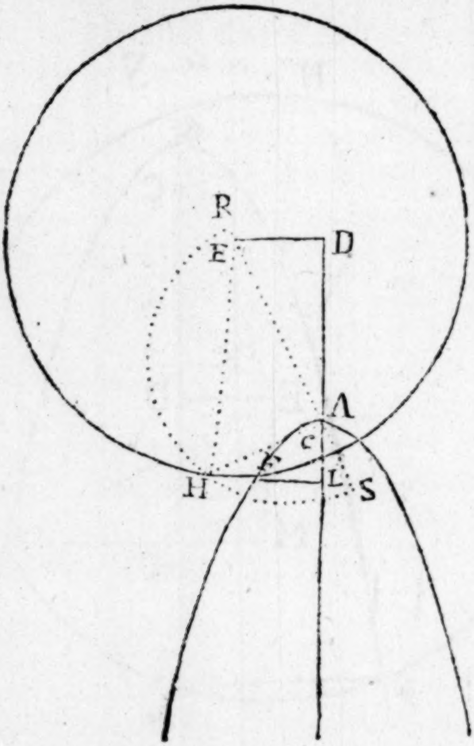
But if the Equation be a *Biquadratick*, there is ſomething farther neceſſary to find the Radius of the Circle. For then the fourth Term r being there, and having a Poſitive Sign, take downwards from A the Vertex of the Parabola on the Line AE, which ſuppoſe drawn $AR = r$ and produce R A, till A S become equal to the Parameter, or $= a = 1$. Then make R S the Diameter of a Circle, and at A erect A H Perpendicularly; it ſhall cut the Semicircle in H: So that H E ſhall be the Radius



Radius of the Circle that is to cut the Parabola. (*See alſo Fig. following.* Where the Point D, is in the Axis produced above the Vertex.)

But if the Quantity *r* happen to have a Negative Sign in the Equhtion; there muſt yet another Circle be deſcribed on A E as a Diameter: In which accommodate A I = to A H the Perpendicular before found; which will find the Point I, through which the Interſecting Circle muſt paſs: And whoſe Radius will be I E, and Center E, as before.

See Fig. in Pag. 84.



And by this means a Circle will be drawn, which will cut the Parabola in 1, 2, 3 or 4 Points, from whence Perpendiculars let fall to the Axis, will be all the poſſible or real Roots of the Equations, Affirmative or Negative: The former of which will be on the ſide of the Axis that the Center E is, when 'tis $+q$, but on the other Side, when it is $-q$.

The Demonſtration of all which he thus very eaſily gives us.

Call

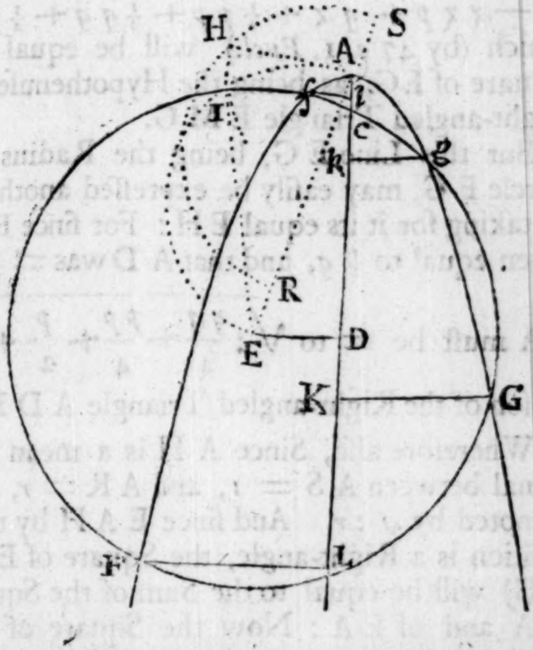
with the Ordinate [illegible] the Curve [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

[illegible]

And for explication [illegible] was [illegible] on the [illegible] from [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] which [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

For [illegible] of all which [illegible] [illegible] [illegible]

Call the Ordinate G K, by this Conſtruction found to be a true Root, by the Name of z, Then 'tis plain the Abſciſſa, A K muſt be zz, becauſe in the Parabola the Ordinate is always a mean Proportional between the Parameter (here ſuppoſe



$a = 1$.) and the Abſciſſa; by the firſt Property of this Figure above demonſtrated. Wherefore if from A K you take A C $= \frac{1}{2}$ (or $\frac{1}{2}a$) and then C D $= \frac{1}{2}p$: The Remainder D K ($=$ E M) will in this Notation, be $zz - \frac{1}{2}p - \frac{1}{2}$. (*See the two former Figures.*) And this Quantity $zz - \frac{1}{2}p - \frac{1}{2}$ Squared, produces $z^4 - zzp - zz + \frac{pp}{4} + \frac{p}{2} + \frac{1}{4}$. And becauſe by the Conſtruction D E or KM $= \frac{1}{2}q$, therefore the whole GM will be in this Notation $= z + \frac{1}{2}q$, whoſe Square is $zz + zq + \frac{qq}{4}$. Add then this and the former Square of DK or EM together, and it makes $z^4 - zzp + qz + \frac{1}{4}pp + \frac{1}{4}qq + \frac{1}{2}p + \frac{1}{4}$; which (by 47 *e.* 1. *Eucl.*) will be equal to the Square of EG, as being the Hypothenuſe of the Right-angled Triangle EMG.

But this Line EG, being the Radius of the Circle FG, may eaſily be expreſſed another way, by taking for it its equal EH: For ſince ED was taken equal to $\frac{1}{2}q$, and that AD was $= \frac{1}{2}p + \frac{1}{2}$, EA muſt be $=$ to $\sqrt{:\frac{qq}{4} + \frac{pp}{4} + \frac{p}{2} + \frac{1}{4}}$, by reaſon of the Right-angled Triangle ADE.

Wherefore alſo, Since AH is a mean Proportional between AS $= 1$, and AR $= r$, it muſt be noted by $\sqrt{:r}$. And ſince EAH by the Suppoſition is a Right-angle, the Square of EH (*i.e.* EG) will be equal to the Sum of the Squares of HA and of EA: Now the Square of EA is $\frac{qq}{4} + \frac{pp}{4} + \frac{p}{2} + \frac{1}{4}$: To which adding the Square of AH $= r$, it will ſtand thus, $\square$ EH $= \frac{1}{4}qq + \frac{1}{4}pp + \frac{1}{2}p + \frac{1}{4} + r$: And ſince EG $=$ EH, theſe two Quantities will be equal, *viz.* $z^4 - zzp + qz + \frac{1}{4}pp + \frac{1}{4}qq + \frac{1}{2}p + \frac{1}{4} = \frac{qq}{4} + \frac{pp}{4} + \frac{p}{2} + \frac{1}{4} + r$: Compare then theſe two,

and

and reject what is common to both, and you will find remain $z^4 + p\,zz - b\,z + r = 0$.

From whence it appears, That the Ordinate G K, which we called z, is the true Root of the Equation propoſed to be conſtructed. *Q. E. D.*

And if you apply this Calculation to all other Caſes of this Rule, changing the Sign + or —, as occaſion requires, you will gain your Deſign after the ſame manner.

Thus far went *Des Cartes* in this Matter; but he conſidering only the Axis of the Parabola, and not thinking what might be done by the other Diameters or Parallels to the Axis, could not this way conſtruct either *Cubick* or *Biquadratick Equations*, till he had firſt ejected or taken away the ſecond Term: Which to effect is a tedious and troubleſome Operation.

All which our Famous Mr. *Thomas Baker* Rector of *Nympton*, in the County of *Devon*, well conſidering, and having withal ſeen how near *Schooten* was of gaining the Point, (who drew a Parallel to the Axis without the *Curve*, and by that means conſtructed *Cubicks*, without taking away the ſecond Term. See his Comment on *Des Cartes* third Book, Page 328.) He thought of drawing a Parallel to the Axis within the Figure; whereby, and by the help of Mr. *Strode*'s Property above mentioned, viz. *That the Parameter is to the Sum of any two Ordinates* : : *as their Difference is to the Difference of their Abſciſſæ* : He found he could conſtruct all ſorts of Equations, not exceeding four Dimenſions; whether all their Terms were there, or whether the ſecond third or fourth Term, or all of them were wanting. And that he could find the Center of a Circle which would

cut

cut the Parabola in as many Points as the Equation had real Roots: Which real Roots, would now be Perpendiculars let fall from thoſe Interſected Points of the *Curve*, to the ſaid Diameter or Parallel to the Axis. Of this he gives abundance of Examples and Caſes, with their Demonſtrations, in his *Geometrical Key*, or *Gate of Equations unlock'd.* And becauſe the chief or rather only difficulty, lies in finding the Center of a Circle which ſhall Interſect the Parabola in the Points required: He gives us for this purpoſe, what he very properly calls his *Central Rule*, which is a Rule conſiſting of two Parts: By the former of which he determines the Point D in the Parallel to the Axis, from whence the Perpendicular D E is to be erected: And by the latter, the length of that Perpendicular, whereby he finds the Point E, the Centre of the Circle required. *See the following Figure in Page* 89.

His Central Rules are theſe.

$$\left.\begin{array}{l} \text{I. } \dfrac{L}{2} + \dfrac{pp}{8L} + \dfrac{q}{2L} = b = AD. \\[2ex] \text{II. } \dfrac{p}{4} + \dfrac{ppp}{16LL} + \dfrac{q}{4LL} + \dfrac{r}{2LL} = d = DE. \end{array}\right\}$$

And becauſe L the *Latus Rectum* or *Parameter* of the Parabola is equal to 1, it may be contracted into this Form.

$$\text{I. } \frac{1}{2} + \frac{pp}{8} \pm \frac{q}{2} = b = AD.$$

$$\text{II. } \frac{p}{4} + \frac{ppp}{16} + \frac{pq}{4} + \frac{r}{2} = d = ED.$$

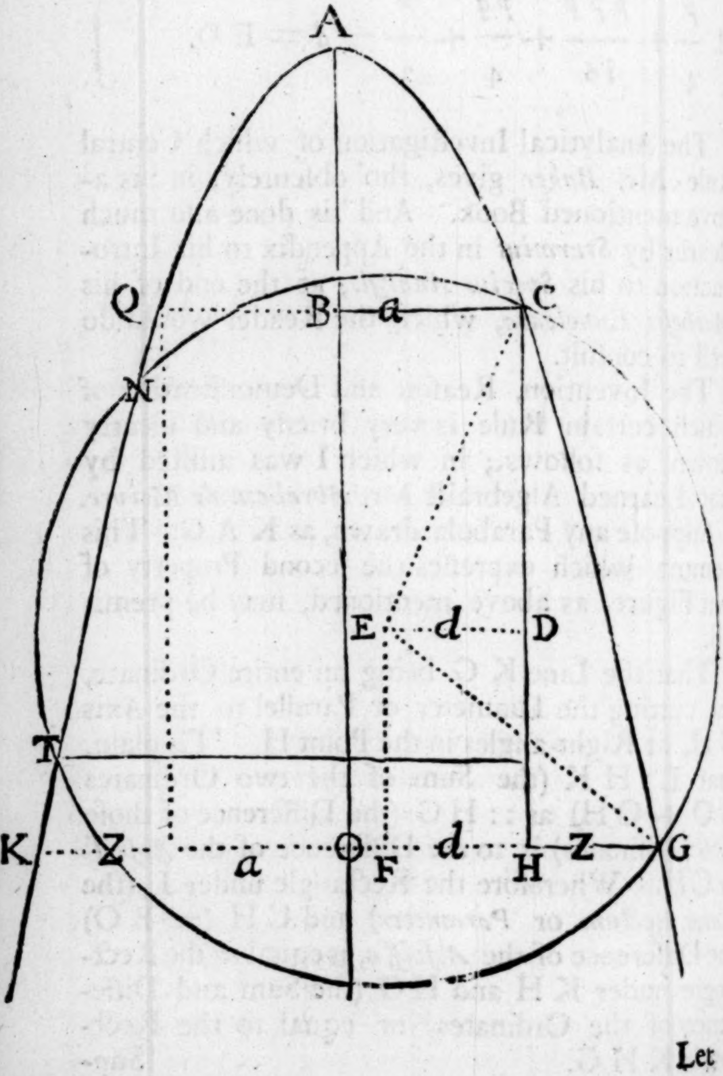
The Analytical Inveſtigation of which Central Rule Mr. *Baker* gives, tho' obſcurely, in his above mentioned Book. And 'tis done alſo much clearer by *Sturmius* in the Appendix to his Introduction to his *Specius Analyſis*, at the end of his *Matheſis Enucleata*, which the Reader would do well to conſult.

The Invention, Reaſon and Demonſtration of which certain Rule is very briefly and clearly ſhown, as follows; in which I was aſſiſted by that Learned Algebraiſt Mr. *Abraham de Moivre*.

Suppoſe any Parabola drawn, as K A G. This Lemma which expreſſes the ſecond Property of that Figure, as above mentioned, may be premiſed.

That the Line K G being an entire Ordinate, and cutting the Diameter or Parallel to the Axis C H, at Right-angles in the Point H. 'Tis plain, that L : H K (the Sum of the two Ordinates G O + O H) as : : H G (the Difference of thoſe two Ordinates) is to the Difference of the *Abſciſſæ* C H. Wherefore the Rectangle under L, (the *Latus Rectum* or *Parameter*) and C H (= B O) the Difference of the *Abſciſſæ*, is equal to the Rectangle under K H and H G (the Sum and Difference of the Ordinates) or equal to the Rectangle K H G.

Suppoſe then in the Figure annexed, Mr. *Baker's* L, the Parameter or *Latus Rectum* belonging to this Parabola, to be called $p = 1$.



Let

Let $CB = a$. $GH = z$ the Root ſought; then will be
$CD = b$. $FG = z + d$. And
$DE = d$. $HK = 2a + z$.

Wherefore by the Lemma, or ſecond Property of the Parabola $p \times$ by $CH = 2az + zz$.

Wherefore $CH = \frac{2az + zz}{p}$, and EF ($= DH$) $= \frac{2az + zz}{p} - b$; but (by 47 e. 1 *Eucl.*) FEq; $+ FGq$; $= EGq$; $= ECq = EDq + DCq$. That is, in this way of Notation, $\frac{4aazz + 4az^3 + z^4}{pp} - \frac{4baz - 2bzz}{p} + bb + zz + 2zd + dd = bb + dd$.

Strike out $bb + dd$ being common to both Sides of the Equation, and it will ſtand thus, $\frac{4aazz + 4az^3 + z^4}{pp} - \frac{4baz - 2bzz}{p} + zz + 2dz = 0$. After this, if you Multiply the laſt Equation by pp, and then divide the Product by z, you will bring it to this Form, $4aaz + 4azz + zzz - 4pba - 2pbz + ppz + 2ppd = 0$.

Or, to reduce it to a more regular Form, where the Terms ſhall be ranged in their proper Order: Let it ſtand thus,

Z

$$\begin{array}{l} Z^3 + 4azz + ppz - 4bpa \\ \quad + 4aaz + 2ppd = 0 \\ \quad - 2pbz \end{array}$$

Or, if you had called the whole Line C Q, by the Name of *a*, as *Baker* calls it *p*, the Equation would ſtand thus.

$$\begin{array}{l} Z^3 + 2aZZ + ppz - 2bpa \\ \qquad\qquad + aaz + 2pbd = 0 \\ \qquad\qquad - 2bpz. \end{array}$$

And this is a Cubick Equation produced, having all its Parodick Degrees, or Terms, and one of whoſe true Roots is apparently z, a Right-line let fall from G, the Point of Interſection of the Circle and the Parabola, to the Diameter or Parallel to the Axis C H.

Now if you will compare this Equation Member by Member with one given to be ſolved in the common Form, as ſuppoſe $Z^3 + mZZ + rZ - S = 0$, you will find all things reſpectively equal. Deſcribe any *Parabola*, as K A G in the laſt Figure, where apply Q C at Right-angle to the Axis $= \frac{1}{2} m$. Then compare the *Coefficients* in the two next correſponding Terms in both Equations, and you will have $pp + aa - 2pb$ in the former, $= r$ in the latter or common Equation. This is (becauſe 2 a the Coefficient of the ſecond Term in one, is equal to m in the other Equation, and conſequently $a = \frac{1}{2} m$) $pp + \frac{1}{4} mm - 2pb = r$.

Where-

Wherefore $b = \frac{pp + \frac{1}{4}mm - r}{2p} = \frac{1}{2}p + \frac{mm}{8p} + \frac{\frac{1}{2}r}{p}$: Which is the Rule to find the length of C D, or to determine the Point D, nearly the ſame, changing only the Letters with Mr. *Baker*'s firſt Rule, *viz.* $\frac{L}{2} + \frac{pp}{8L} + \frac{q}{2L} = b =$ A D.

Then again, compare the next two correſponding and equal Terms in both Equations together, and you will have $2pbd - 2bpa = S$; wherefore $d = \frac{S + 2bpa}{2pb}$; which being all a known Quantity, the length of the Line E D is known; and conſequently the Central Point E is found, on which the Circle is to be deſcribed with the Radius E C, or E G.

For in every *Cubick Equation*, where are all the Terms, the Circle will paſs thro' the Point C, the Vertex of the Diameter or Parallel to the Axis. And the Circle will cut or touch the Parabola in as many Points as the Equation hath *real Roots*, which will be *Perpendiculars*, let fall from thoſe Points to the Diameter C H. And which of them are Poſitive or Affirmative, and which are Negative, hath been ſhewn above.

But if the Equation had been a *Biquadratick*, the Circle will not paſs thro' the Vertex of the Diameter, but thro' another Point, to be found according to the Rule above mentioned in *Des Cartes* his Conſtruction, which is to ſet

$\sqrt{\frac{S}{pp}}$ (if S repreſent the Abſolute Number or fifth Term in ſuch an Equation, and have a Negative Sign) at Right-angles to E C, on the Vertex of the Diameter; but if it be + S, then that Line $\sqrt{\frac{S}{pp}}$ muſt be inſcribed in another Semicircle made on the Line E C; which being ſet from the Vertex of the Diameter C, will give in the Periphery of that Semicircle on E C, a Point thro' which the Interſecting Circle required ſhould paſs.

And thus are *Cubick* and *Biquadratick Equations* conſtructed, having all their Terms. But ſuch as want the ſecond, third, or any other Term or Terms, may be as well conſtructed this way, only by leaving out of the Central Rule, that Part of it which belongs to ſuch *wanting Terms*, and going on as is above ſhew'd with the reſt.

If you would ſee how the Central Rule is Inveſtigated in the laſt Term of a *Biquadratick*, (as well as here in a Cubick) according to *Baker's* Method.

Suppoſe Q C as before = *a*. Then if z in the Figure be taken for one of the real Roots of a *Biquadratick*, a regular Equation will be produc'd in this Form.

$$\left.\begin{array}{llll} Z^4. & 2az^3. & aazz. & 2pabz \\ & & 2pbzz. & 2pbdz - S \\ & & ppzz. & \end{array}\right\} = 0.$$

Compare

Compare this, as is above ſhew'd, with any perfect *Biquadratick* in the Common Form: As ſuppoſe, $Z^4.\ mz^3.\ qzz.\ rz.\ S = 0$. Firſt, make $QC = \frac{1}{2}m$; which is equal every where to a in the other Equation, and putting therefore half m inſtead of a, that will ſtand thus.

$$\left.\begin{array}{ll} Z^4 + mz^3 + \dfrac{mm}{4}zz - pmbz & \\ & -S \\ -2pbzz + 2pbdz & \\ +\ ppzz & \end{array}\right\} = 0.$$

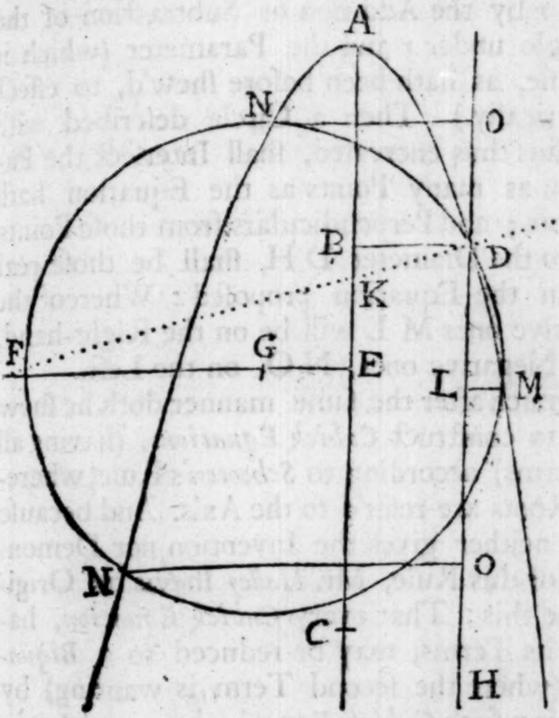
The ſecond Term being expreſs'd by QC, go on to compare the third Term in each, ſo you will have $\frac{mm}{4} - 2pb + pp = q$. Wherefore, $pp + \frac{mm}{4} - q = 2pb$. And conſequently dividing all by $2p$ $\frac{p}{2} + \frac{mm}{8p} - \frac{q}{2p} = b = CD$, which is the firſt part of the Central Rule.

And ſince $-pmb + 2pbd = r$, therefore $pmb + r = 2pbd$. But $b = \frac{p}{2} + \frac{mm}{8p} - \frac{q}{2p}$: Wherefore ſubſtituting this inſtead of b in the laſt Equation; it will be $\frac{ppm}{2} + \frac{mmm}{8} + \frac{mq}{2} + r = 2pbd$: And conſequently dividing all by $2pb$, it will ſtand thus; $\frac{m}{4} + \frac{m^3}{16pp} + \frac{mq}{4pp} + \frac{}{2pp}$ $= d =$ D E; which is the ſecond part of the Central Rule in a Compleat *Biquadratick*.

That Excellent Mathematician Capt. *Edmund Halley*, in his *Philoſophical Tranſactions*, N. 188. hath a peculiar Diſſertation on this Subject: *Of the Conſtruction of Solid Problems*; or, *Of Equations of the third or fourth Power.* In which he not only gives the Reaſon and Foundation of Mr. *Baker*'s Rule, gets rid of the Intricate Cautions of *Baker*, in reference to the Signs, *&c.* but he gives alſo a new Conſtruction of thoſe Equations which is very eaſie, and ſhort: And which therefore I ſhall now annex to what hath been already done.

Let any *Cubick* or *Biquadratick Equation*, having all its Terms, be given to be conſtructed in one of theſe Forms, $Z^3.\ bzz.\ apz.\ aaq = 0.$ or $Z^4.\ bzzz.\ apzz.\ aapz.\ a^3.\ r = 0.$ to which it will be capable of being reduced. Then deſcribe a *Parabola*, as N A M, whoſe *Parameter* or *Latus Rectum* let be a: Its Vertex A, and its Axis A B C. Then apply at Right-angles to the Axis B D $= \frac{1}{4} b$, the ſecond Term in the Equation; and thro' D draw D H Parallel to the Axis, and let it be placed on the Left-hand, if b have a Negative Sign; but on the Right-hand if it be $+ b$. In the Line A B *continued* downwards towards B, take B K $= \frac{1}{2} a$: and then draw the Infinite Line F K D, take K C $= 2$ A B, always downwards from K; and if the Quantity p have a Negative Sign, take alſo the ſame way C E $= \frac{1}{2} p$; but on the contrary, take it upwards if it

be



be + *p*; then at the Point E thus found (or from the Point C, if *q* be *wanting*) let there be erected E F, perpendicular to the Axis, which ſhall cut the Line D K, when produc'd in the Point F. Which Point F, if the Quantity *q* be wanting, ſhall be the Centre of the Circle required; but if *q* be in the Equation, and have a Negative Sign, then ſet towards the Right-hand F G = $\frac{1}{2}q$; but if *q* have a Poſitive Sign, F G muſt be ſet on the Left-hand of F, on E F produced that way. So ſhall G be the Centre of the Circle required, and G D its Radius, if the Equation be a *Cubick*. But if it be a *Biquadratick*, then the Square of C D muſt be augmented if it be $-r$, or diminiſhed if it be $+r$ by the Addition or Subtraction of the Rectangle under r and the Parameter (which is very eaſie, as hath been before ſhew'd, to effect Geometrically.) Then a Circle deſcribed with the Radius thus encreaſed, ſhall Interſect the Parabola in as many Points as the Equation hath real Roots; and Perpendiculars from thoſe Points let fall to the Diameter D H, ſhall be thoſe real Roots in the Equation propoſed: Whereof the Affirmative ones M L will be on the Right-hand, and the Negative ones, N O, on the Left.

And much after the ſame manner doth he ſhew us how to conſtruct *Cubick Equations*, (having all their Terms) according to *Schooten*'s Rule, whereby the Roots are refer'd to the Axis. And becauſe *Schooten* neither gives the Invention nor Demonſtration of this Rule, Mr. *Halley* ſhews its Original to be this: That every *Cubick Equation*, having all its Terms, may be reduced to a *Biquadratick* (where the ſecond Term is wanting) by multiplying ſuch *Cubick Equation* by $z - b = 0$, if it be $+b$, or $Z + b = 0$, if it be $-b$. Which new produced Equation ſhall have the ſame Roots as the *Cubick* had, and alſo one more equal to $\mp b$, according as the Sign of b was in the Equation. As for Inſtance, Let this Equation $Z^3 - zzb + apz - aaq = 0$, be propoſed to be conſtructed.

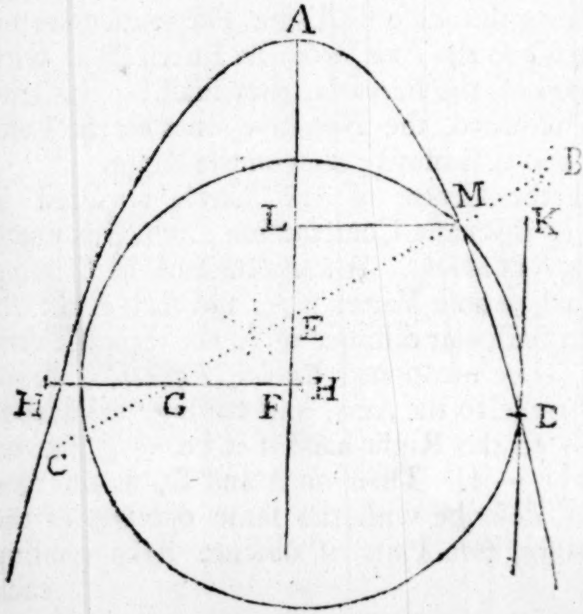
This Multiplied by $z + b$ (becauſe b hath a Negative Sign) makes $Z^4 - z^3 b + apzz + aaqz + z^3 b - zzbb + apzb + aaqb$, which Equation, when conſidered, will want its ſecond Term, becauſe $-Z^3 b$ and $+Z^3 b$, do deſtroy one another. So it will ſtand thus,

Z^4

$$z^4 * + apzz + aaqz$$
$$- bbzz + abzp + aaqb = 0.$$

Where the Coefficients of the third Term, *viz.* $-bb+ap$, do give $-\frac{bb}{2a}+\frac{1}{2}p$, to be taken inſtead of $\frac{1}{2}p$, or C D, in *Des Cartes* Conſtruction: And the Coefficients of the fourth Term, $aaq+abp$ give $\frac{1}{2}q+\frac{bp}{2a}$ to be taken inſtead of $\frac{1}{2}q$ or D E, by which means the Center of the Circle is determined; and becauſe one of the Roots of the New Equation, *viz.* $+b$, or $-b$ is given, the Radius of the Circle, or Point in the Circumference will be known alſo. A Circle then being thus deſcribed, and Perpendiculars being let fall to the Axis from its Interſection with the *Curve* of the *Parabola*, they ſhall be the true Roots required, the Negative ones on the Left-hand, and Affirmative ones on the Right.

And the Centre of the Circle required is found by this eaſie Conſtruction; which is much the beſt for *Cubicks*. A *Parabola* as A M D being deſcribed, whoſe Vertex is A, and Axis A H: At the Perpendicular diſtance of b, the ſecond Term in the above mentioned *Cubick Equation*, draw K D Parallel to the Axis, and cutting the Parabola in D, on the Right-hand if it be $+b$, but on Left it be $-b$. Then on A and D, as on two Centres, deſcribe with the ſame opening of the Compaſſes, two Pairs of obſcure Arks croſſing each other, as you ſee in the following Figure, and thro' which is to be drawn the Infinite Line B C at Right-angles with the ſuppoſed Line A D, and cutting the Axis in the Point E. Then ſet (downward from E if it be $-p$, but upward towards A if it be $+p$) $EF = \frac{1}{2}p$. And from F (or from E if p be wanting) erect the Perpendicular Line F G, cutting the Infinite one B C, in the Point G. Laſtly, Produce $GH = \frac{1}{2}q$ (towards the Right-hand if it be $-q$, but towards the Left-hand, if it be $+q$) and then ſhall H be the Centre of the Circle required, and H D the Radius. Which Circle ſhall Interſect the Parabola in the Points M and D, whence Perpendiculars let fall to the Axis, ſhall be the true Roots: And both Affirmative, becauſe on the Right-ſide. The Demonſtration and Reaſon of which, is evident from what hath been above deliver'd.



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The Learned Mr. *Halley* in *Philoſ. Tranſactions*, N. 190. hath an excellent Diſcourſe of the Number of Poſſible Roots in a *Solid*, or *Biquadratick Equation*; and alſo of their Limits: Where he ſhews, from the Principles of the above written Method of Conſtructions, That ſince a Circle interſecting a Parabola muſt do it, either in two Points or four: Therefore in *Biquadratick Equations*, there will be either two or four real Roots, Affirmative or Negative. And that if it happens that the Circle touch the Parabola only in any Point, and do not cut it: 'Tis then an Indication of the Equality of two Roots having the ſame Sign.

But in *Cubicks* of all ſorts, and however *Adfected*, there is either but one, or elſe three Poſſible Roots, ſuppoſing you allow Negative ones as ſuch.

In *Biquadraticks*, if the laſt Term r have a Negative Sign, there are always either two or four Roots. But if it be $+ r$, and alſo that $\sqrt{:\mathrm{GD}^{q} - ar}$, *(See the laſt Figure ſave one)* be ſo ſmall that the Circle deſcribed with that Radius on the Centre G, cannot touch or cut the Parabola in any Point: Then is that Equation utterly Impoſſible; ~~and~~ is Explicable by no poſſible Root, either Affirmative or Negative. By what means he attains to the knowledge of theſe Rules and Limitations, the Reader may find there at large, where he fully Illuſtrates all things with proper Examples.

OF

SURD ROOTS.

WHEN any Number or Quantity hath its Root propoſed to be Extracted, and yet is not a *true Figurate Number of that kind:* That is, if its Square Root being demanded, it is not a *true Square* : If its Cube Root being requir'd, it ſelf be not a *true Cube*, &c. then 'tis impoſſible to aſſign, either in whole Numbers or Fractions, any exact Root of ſuch Number propoſed. And whenever this happens, 'tis uſual in *Mathematicks*, to mark the required Roots of ſuch Numbers or Quantities, by prefixing before it the proper Mark of Radicallity, which is $\surd$: Thus $\surd$: 2, ſignifies the Square Root of 2, and $\sqrt[3]{}$: 16, or $\surd$: (3) 16, ſignifies the *Cubick Root* of 16 : Which Roots, becauſe they are impoſſible to be expreſſed in Numbers exactly (for no effable Number, either Integer or Fraction Multiplied into it ſelf can never produce 2 ; or being Multiplied Cubically, can ever produce 16) are very properly call'd *Surd Roots*.

There is alſo another way of *Notation* now much in uſe, whereby Roots are expreſſed, without the Radical Sign, by their *Indexes* : Thus, as x^2, x^3,

Of

Surd roots.

1 Reduce $a=4$ to ye form of $b=\sqrt[2]{17}$ $aa=\sqrt[2]{16}$.

Reduce $\frac{\sqrt[2]{3}}{6}$ to ye form of $\sqrt[2]{\frac{2}{24}}$. $\sqrt[2]{\frac{3}{36}}$.

2 Reduce these heterogeneal surds $\sqrt[2]{:ee}$ & $\sqrt[3]{:uu}$ to others that are homogeneal.

$\sqrt[6]{:eeeeee}$ & $\sqrt[6]{:uuuu}$

Reduce these surds $\sqrt[2]{:50}$ & $\sqrt[6]{:100}$ to others that are homogeneal.

$\sqrt[6]{\underset{\wedge}{1}\overset{2}{,}5\underset{\wedge}{0}^{00}}$ & $\sqrt[6]{100}$.

3 Reduce this surd $\sqrt[2]{:25ab}$ to it's lowest terms possible.

$5\cdot\sqrt[2]{:ab}$.

Reduce this surd $\sqrt{:28}$ to it's lowest terms possible $2\sqrt[2]{:7}$.

4 Find whether or no these two roots $\sqrt[2]{:20}$ & $\sqrt[2]{:10}$ be Commensu-

ſurable. $\sqrt[2]{}$:2, & $\sqrt[2]{}$:1 [illegible] The root of 2 being inexpreſsible by any true number, theſe ſurds are not commenſurable.

Are theſe ſurds $\sqrt[2]{}$:10 & $\sqrt[2]{}$:90 commenſurable? Yes. For the root of 1 is 1 & the root of 9 is 3.

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x^5, ſignifie the Square, Cube and 5th Power of x; ſo $x^{\frac{1}{2}}$, $x^{\frac{1}{3}}$, $x^{\frac{1}{5}}$, *&c.* ſignifie the Square Root, Cube Root, *&c.* of x. The Reaſon of which is plain enough, for ſince $\sqrt{}:x$ is a Geometrical mean Proportional between 1 and x. So half is an Arithmetical mean Proportional 0 and 1, and therefore as 2 is the Index of the Square of x, half will be the proper Index of its Square Root, *&c.*

Obſerve alſo, that for Convenience or Brevity ſake, Quantities or Numbers which are not *Surds*, are often expreſſed in the Form of *Surd Roots:* Thus, $\sqrt{}:4$, $\sqrt{}:\frac{9}{4}$ $\sqrt[3]{}:27$, *&c.* ſignifie 2, $\frac{3}{2}$, 3, *&c.*

But altho' theſe *Surd Roots* (when truly ſuch) are inexpreſſible in Numbers, they are yet capable of Arithmetical Operations (ſuch as *Addition, Subtraction, Multiplication, Diviſion,* &c. which how readily to perform, the Algebraiſt ought not to be Ignorant.

Surds are either *Simple,* which are expreſſed by one ſingle Term; or elſe *Compound,* which are formed by the Addition or Subtraction of Simple Surds: As $\sqrt{}:5+\sqrt{}:2:\sqrt{}:5-\sqrt{}:2$, or $\sqrt[3]{}:\overline{7+\sqrt{}:2}$: Which laſt is called an *Univerſal Root:* And ſignifies the *Cubick Root* of that *Number which is the reſult of adding 7 to the Square Root of 2.*

The Arithmetick of Surds conſiſts of theſe principal Parts.

I. *To reduce Rational Quantities to the Form of any Surd Roots aſſigned.*

Which is perform'd by Involving the Rational Quantity according to the Index of the Power of the Surd, and then prefixing before it the Radical Sign of the Surd propoſed.

Thus to reduce $a = 10$ to the Form of $\surd : 15, = \surd : b$, you muſt Square $a = 10$; and prefixing the Sign, it will ſtand thus, $\surd : aa = \surd : 100$, which is in the Form of the Surd deſired. So alſo if 3 were to be brought to the Form of $\sqrt[4]{} : 12$, you muſt raiſe 3 up to its fourth Power, and then prefixing the Note of Radicality to it, it will be $\sqrt[4]{} : 81$, or $81^{\frac{1}{4}}$, which is in the ſame Form with $\sqrt[4]{} : 12$.

And this way may a Simple Surd Fraction, whoſe Radical Sign refers only to one of its Terms, be changed into another, which ſhall reſpect both Numerator and Denominator. Thus, $\frac{\surd : 2}{5}$ is reduced to $\surd : \frac{2}{25}$ and $\frac{5}{\sqrt[3]{} : 4}$, to $\sqrt[3]{} : \frac{125}{4}$ where the Radical Sign affects both Numerator and Denominator alike.

II. *To*

II. *To Reduce Simple Surds, having different Radical Signs* (which are called *Heterogeneal Surds;*) *to others that may have one common Radical Sign, or which are Homogeneal.*

Divide the Index of the Powers of the Surds by their greateſt common Diviſor, and ſet the Quotients under the Dividends; then Multiply thoſe Indexes croſs-ways by each others Quotients, and before the Products ſet, the common Radical Sign $\sqrt{}$: with its proper Index: Then Involve the Powers of the given Roots Alternately, according to the Index of each others Quotient, and before thoſe Products prefix the common Radical Sign before found.

To Reduce $\sqrt[2]{}:aa$ and $\sqrt[4]{}:bb$.

$$\begin{array}{ccc} 2)\ \sqrt{}:aa & & 2)\ \sqrt[4]{}:bb \\ 1 & \times & 2 \\ \sqrt[4]{}:bb & & \sqrt[4]{}:aaaa. \end{array}$$

To Reduce $\sqrt{}:5$ and $\sqrt[4]{}:7$.

$$\begin{array}{ccc} \sqrt[2]{}\,5 & & \sqrt[4]{}:7 \\ 1 & \times & 2 \\ \sqrt[4]{}:25 & & \sqrt[4]{}\,2401. \end{array}$$

III. *To Reduce Surds to their loweſt Terms poſſible.*

Divide the Surd by the greateſt Square, Cube, Biquadrate, *&c.* or any other higher Power, which you can diſcover is contained in it, and will Meaſure it without any Remainder, and then prefix the Root of that Power before the Quotient, or Surd ſo divided, and this will produce a new Surd of the ſame value with the former, but in more ſimple Terms. Thus, $\surd : 16\,aab$, by dividing by $16\,aa$ and prefixing the Root $4\,a$, will be reduced to this $4\,a \surd : b$ and $\surd : 12$ will be depreſs'd to $2 \surd : 3$. Alſo $\sqrt[3]{\ } : c\,b^3 r$, will be brought down to $b \sqrt[3]{\ } : c\,r$. And this Reduction is of great uſe whenever it can be perform'd: But if no ſuch Square, Cube, Biquadrate, *&c.* can be found for a Diviſor; then you muſt find out all the Diviſors of the Power of the Surd propos'd; and then ſee whether any of them be a Square, Cube, *&c.* or ſuch a Power as the Radical Sign denotes; and if any ſuch can be found, let that be uſed in the ſame manner as is above ſaid, to free the Surd Quantity in part from the Radical Sign. Thus, If $\surd : 288$ be propos'd; among its Diviſors will be found the Squares 4, 9, 16, 36, and 144, by which if 288 be divided, there will ariſe the Quotients 72, 32, 18, 8 and 2, wherefore inſtead of $\surd : 288$, you may put $2 \surd : 72$, or $3 \surd : 32$, or $4 \surd : 18$, or $6 \surd : 8$, or laſtly, $12 \surd : 2$, and the ſame may be done in Species.

IV. To

IV. *To find whether two Surd Roots given are Commenſurable or not.*

Thoſe are called *Commenſurable Surds*, which are to one another as Number to Number, as one Rational Quantity to another; or which are, when reduced to their leaſt Terms, true Figurate Quantities of their own kind.

To diſcover therefore, whether they are ſuch or not; If the Surds are of different kinds (or *Heterogeneal Surds* as ſome call them) they muſt firſt be reduced to one kind, and then divided ſeverally by their greateſt common Meaſure, for if then there will come out Rational Quotients, the firſt Surds are *Commenſurable*; but if the Quotients are Irrational, or Surd Numbers or Quantities, then the propoſed Surds are Incommenſurable.

V. gr. To examine whether $\surd : 12$, and $\surd : 3$, are Commenſurable Surds? They being Homogeneal, I divide them ſeverally by their greateſt Common Diviſor, which is $\surd : 3$; and the Quotients are $\surd : 4$, and $\surd : 1$, that is, 2 and 1. Wherefore, ſince 2 and 1, are Rational Numbers, I ſay that $\surd : 12$ and $\surd : 3$, are Commenſurable Surds; or are to one another, as 2 to 1, which is very plain; for no doubt 12 : 3 : : as 4 : 1, and 'tis plain, that as Squares are to one another, ſo are their Roots: Wherefore 12 : 3, as $\surd : 12$, $\surd : 3$, that is, as $\surd : 4$, $\surd : 1$, or as 2 to 1.

Whenever two Surds are divided by one common Diviſor, (tho' not the greateſt) if their Quotients come out Rational, or are to one another as Number to Number, thoſe Surds are certainly Commenſurable.

If Fractional Surds were given, not having a common Denominator, they muſt firſt be reduced to their ſmalleſt common Denominator, and then if their Numerators are commenſurable, you may conclude the firſt Surd Fractions were ſo.

But if either the Numerators or Denominators of two Surds proper Fractions, or mixt Numbers in the Form of Fractions (neglecting the Radical Sign) be Powers of that kind which the Radical Sign expreſſes, then they will need no Reduction: For if their Numerators or Denominators are Commenſurable, the whole Surd Fractions propoſed are certainly ſo. Thus, If it were enquired, Whether $\sqrt{}::\frac{50}{16}$ and $\sqrt{}:\frac{72}{25}$ are Commenſurable Surds; becauſe 16 and 25 are Squares, or ſuch Powers as the Radical Sign expreſſes or Denotes, omitting the Sign $\sqrt{}$: you need only compare the Numerators $\sqrt{}:50$, and $\sqrt{}:72$; which being divided by their greateſt common Diviſor $\sqrt{}\,2$, the Quotients will be 5 and 6 (*i. e.* $\sqrt{}:25$ and $\sqrt{}:36$.) Wherefore the given Surds are Commenſurable, and are to one another, as $\frac{5}{4}$ to $\frac{6}{5}$; and conſequently, by the precedent Rule, may be expreſſed thus, $\frac{5}{4}\sqrt{}:2$ and $\frac{6}{5}\sqrt{}:2$.

For an Inſtance in *Species*: Suppoſe that it were enquired whether $\sqrt{}:27aa$, and $\sqrt{}:12aa$ were Commenſurable Surds? Divide each by the greateſt common Diviſor, $\sqrt{}:3aa$: And the Quotients $\sqrt{}:9$ and $\sqrt{}:4$; that is, 3 and 2, are Rational Numbers; and conſequently, the propoſed Surds are Commenſurable.

Multiplication

Multiply $\sqrt[3]{}:9$ by $\sqrt[3]{}10$

$\sqrt[3]{}:90$.

Multiply $\sqrt[2]{}:11$ by $\sqrt[4]{}:15$

$\sqrt[4]{}:1815$.

Multiply $\sqrt[2]{}:8$ by 4, & y^e product will be $\sqrt[2]{}:128$, or $4\sqrt[2]{}8$.

Square $\sqrt[2]{}:11$, 11.

Multiplication of Simple Surd Roots.

IF the Surds propoſed, be of the ſame kind, Multiply them one by another, and prefix the common Radical Sign to the Product; but if the Surds are Heterogeneal, or of different kinds, they muſt be reduced firſt, (according to *Rule* 2.) to Surds having the ſame Radical Sign.

Thus to Multiply $\sqrt{}:7$ by $\sqrt{}:8$, the Product will be $\sqrt{}:56$.

For ſince in all Multiplication, as 1 is to one Factor, ſo is the other to the Product; therefore here $\sqrt{}:1:\sqrt{}:7::\sqrt{}:8:\sqrt{}:56$. Wherefore $1:7::8:56$, that is, 56 is the true Square of $\sqrt{}:56$, and $\sqrt{}:56$, the true Root of $7\times8=56$.

Other Examples.

I. If $\sqrt{}:8$ were to be Multiplied into $\sqrt[3]{}:4$, becauſe they are not Homogeneal Surds, they muſt be reduced to ſuch by *Rule* 2, and then they will ſtand thus; $\sqrt[6]{}:512$ $\sqrt[6]{}:16$, which being Multiplied into each other, and the common Radical Sign prefix'd, will make $\sqrt[6]{}:8192$: And thus the $\sqrt[6]{}:27$ Multiplied by $\sqrt{}:9$, when reduced, and rightly multiplied, produces $\sqrt[6]{}:531441$.

II. When a Surd is to be Multiplied by a Rational Quantity, that Rational Quantity ought firſt to be reduced to a Surd of like Nature with the true Surd. But 'tis oftentimes convenient only to connect them together, by prefixing the Rational Quantity to the Left-hand of the Surd. As ſuppoſe $\sqrt{}:27$ were to be Multiplied by 6, the Product may commodiouſly be expreſſed thus, $6\sqrt{}:27$, and ſo if $\sqrt[4]{}:9$ were to be Multiplted by 10, it will ſtand thus, $10\sqrt[4]{}:9$.

III. And when two Rational Quantities are thus prefix'd to two Surds of the ſame kind, you may find the Product of them by Multiplying the Rational Part by the Rational, and the Surd part by the Surd, then thoſe joined together will be the Product required. Thus $6\sqrt[3]{}:7$ Multiplied by $5\sqrt[3]{}:3$ Produces $30\sqrt[3]{}:21$.

IV. If any Surd Root be to be Multiplied into it ſelf or *Iuvolved*, according to the Index of its proper Power, you need only caſt away the Radical Sign, and then the Quantity or Number remaining is always the Square, Cube, or other Power required; and will always be Rational. Thus the Square of $\sqrt{}:11$ is 11. The Cube of $\sqrt[3]{}:30$, is 30; alſo $2\sqrt{}:3$ Multiplied by $8\sqrt{}:3=48$, and $3\sqrt{}:5$, Multiplied by $2\sqrt{}:5=30$.

V. And

V. And if the Index of the Power be any even compound Number greater than two, and 'tis required to ſquare ſuch a Surd: There need only a Radical Sign, whoſe Index is half the former, be prefix'd to the Quantity, inſtead of the former Compound one, and it is done. *V. gr.* Suppoſe you would *Square* this Surd, $\sqrt[4]{\ }: 12$; becauſe the Index 4 is compounded of 2 and 2; $\sqrt{\ }: 12$ is the true Product, or the true Square of the Surd Root $\sqrt[4]{\ }: 12$, ſo alſo the Square of $\sqrt[6]{\ }: 10$, is $\sqrt[3]{\ }: 10$.

But when a Simple Surd Quantity, whoſe Radical Sign hath for its Index ſome Ternary Number greater than 3, as 6, 9, *&c.* And 'tis required to Involve this Surd Cubically. Then only prefix before the Quantity a Radical Sign with an Index, which is one third of the former, and 'tis done. Thus, If $\sqrt[6]{\ }: 64$ were to be Cubed, it will be $\sqrt[2]{\ }: 64$, and the Cube of $\sqrt[9]{\ }: 512$, is $\sqrt[3]{\ }: 512$, *&c.* alſo the Biquadrate of $\sqrt{\ }: 5$, is 25 (as being the Square of the Square of $\sqrt{\ }: 5$.) And the Cube of $\sqrt[6]{\ }: 81$ will $\sqrt{\ }: 81$ or 9.

In the general, to *Square*, *Cube*, &c. any Surd Root, is only to Square or Cube the Power, retaining the ſame Note of Radicality; but 'tis better where it can be done, to take one half, one third part, *&c.* of the Exponent of the Root, as is above ſhewn in the laſt particular Rules. (On the contrary, If you would extract the *Square*, *Cube*, or other Root of any Surd, you muſt double or triple, *&c.* the Exponent of the Radicality. Thus the Square Root of $\sqrt{}:16$, is $\sqrt[4]{}:16$, the Square Root of $\sqrt[3]{}:27$, is $\sqrt[6]{}:27$, *&c.*

Diviſion of Simple Surd Roots.

I. IF the Surds are Similar, *Homogeneal*, or of *the ſame kind*, divide one Number or Quantity by another, and prefix the common Radical Sign to the Quotient: But if they are Heterogeneal, or not of the ſame kind, they muſt be *reduced* before they can be divided. Thus, $\sqrt{}:9.)\ \sqrt{}:576\ (\sqrt{}:64 = 8$. And $\sqrt[4]{}:5\ (\sqrt[4]{}:35\ (\sqrt[4]{}:7$.

The Demonſtration of which General Rule is the ſame as that in Multiplication; for from the Nature of Diviſion, the Diviſor is to Unity :: as the Dividend to the Quotient. Therefore in our firſt Inſtance, $\sqrt{}:9 : \sqrt{}:1 :: \sqrt{}:576 : \sqrt{}\,64$, but as theſe Roots are, ſo will their Squares be: That is, $9 : 1 :: 576 : 64$, and that theſe Numbers are truly Proportional, is apparent; becauſe the Rectangles of the Extreams and Means are equal. Wherefore, $\sqrt{}:9,\ \sqrt{}:1 :: \sqrt{}:576,\ \sqrt{}:64$, and conſequently 64 is the true Quotient.

II. If

II. If any Rational Quantity be to be divided by its Square Root, the Square Root will be the Quotient: For if $a\,b$ be divided by $\surd:a\,b$, the Quotient muſt be $\surd,a\,b$: And if 50 be divided by $\surd:50$, the Quotient will alſo be $\surd:50$. Alſo if any Rational Quantity be to be divided by a Surd, that Rational Quantity muſt firſt be reduced to the Form of a Surd by *Rule* 1.

III. When a Surd Root having a Rational Quantity prefix'd before it, is to be divided by the Surd Part of it, the Quotient will be the Rational Quantity. Thus, If $5\surd:9$, be to be divided by $\surd:9$, the Quotient muſt be 5: As if $5\surd:9$ had been divided by 5, the Quotient would be $\surd:9$.

IV. When the Dividend and Diviſor are the Products of two Rational Quantities Multiplied ſeverally into one common Surd; or when they are Rational Quantities prefix'd before one Common Surd; then divide the Rational part of the Dividend, by the Rational part of the Diviſor; and what reſults is the true Quotient. Thus, If $8\surd:5$ be divided by $2\surd:5$, the Quotient will be 4, and if $8\overset{3}{\surd}:7$ be divided by $4\overset{3}{\surd}:7$, the Quotient will be only 2.

V. But when the Dividend and Diviſor are two Rational Quantities or Numbers prefix'd to two unequal Surds; then you muſt divide, not only as before, the Rational Part of the Dividend by *that* of the Diviſor, but alſo the Surd part; and thoſe two Quotients connected together, ſo as the Rational part ſtand on the Left-hand. are the true Quotient ſought. Thus, If $4\surd : 15$ were to be divided by $2\surd : 5$, the Quotient will be $2\surd : 3$ $(=\surd : 12)$ and if $4\surd : 12$, were to be divided by $3\surd : 2$, the Quotient will be $\frac{4}{3}\surd : 6$.

Addition and Subtraction of Surd Roots.

I. WHen two or more Simple and Equal Surds are to be Added, Multiply one of them by the Number of them all, and the Product is the Sum required. Thus the Sum of $\surd : 5$, and $\surd : 5$ is the $\surd : 20$; becauſe $\surd : 5$ Multiplied by 2, the Number of the Surds, that is by $\surd : 4$, gives $\surd :$ 20 to their Sum. Alſo the Sum of $\overset{3}{\surd} : 7 + \overset{3}{\surd} : 7 + \overset{3}{\surd} : 7$ becauſe the Surds are 3 in Number, is $\overset{3}{\surd} : 189$; becauſe $\overset{3}{\surd} : 7$ Multiplied by 3 (*i. e.*) the $\overset{3}{\surd}$: of 27 makes $\surd : 189$.

II. But if *Unequal* Simple Surds of the ſame kind are to be added together, or if one be to be Subtracted from the other, you muſt firſt try whether they are Commenſurable; and if they be, that is, if when they have been divided by their greateſt

Add $\sqrt[2]{\ }:6$, & $\sqrt[2]{\ }:6$ together $\sqrt[2]{\ }:24$, or $2\sqrt[2]{\ }:6$.

Add $\sqrt[2]{\ }:27$ to $\sqrt[2]{\ }:12$. $5\sqrt[2]{\ }:3$, or $\sqrt[2]{\ }:75$

greateſt common Diviſor, their Quotients are Rational Quantities, then you muſt Multiply the Sum of thoſe Rational Quantities by the ſaid common Diviſor, and the Product will be the Sum of the Surds propoſed: Or if the Difference of thoſe Rational Quotients be Multiplied by the Common Diviſor, then the Product will be the Difference of the given Surds, when the leſs is taken from the greater.

Thus if the *Sum* or *Difference* of theſe two Surds $\surd: 50$ and $\surd: 8$, were required; becauſe they are unequal, I try firſt, whether they are Commenſurable, or not, by Dividing each by the greateſt common Diviſor $\surd: 2$: And the Quotients are $\surd: 25$ and $\surd: 4$, that is, 5 and 2, which are Rational Numbers; and therefore the Surds are Commenſurable: Then their *Sum* 7, or their *Difference* 3, Multiplied by the common Diviſor $\surd: 2$, produces $7\surd: 2$ for the Sum, and $3\surd: 2$ for the *Difference* of the Surds required.

III. If the Commenſurable Surds propoſed, had been Fractions, or Mixt Numbers reduced to the Form of Fractions; they muſt (if they have not one) be reduced to a common Denominator in the leaſt Terms; and then to find out the Rational Quotients, you need only Divide the two New Numerators by their greateſt common Diviſor; and then you muſt go on as above, in Integral Surds.

Thus if the *Sum* and *Difference* of $\surd: \frac{24}{25}$ and $\surd: \frac{2}{3}$ were required: When reduced to a common Denominator, they will be $\surd:\frac{72}{75}$ and $\surd:\frac{50}{75}$, and theſe Divided by their greateſt common Diviſor $\surd:\frac{2}{75}$, the Quotes are $\surd:\frac{36}{75}$ and $\surd:\frac{25}{75}$, or $6\surd:\frac{2}{75}$ and $5\surd:\frac{2}{75}$, whoſe Sum is $\surd:11\frac{2}{75}$, and their *Difference* $1\surd:\frac{2}{75}$.

IV. If the Simple Surds given to be Added or Subtracted are *Incommenſurable*, then they can only (generally ſpeaking) be Added or Subtracted by the Signs + and —: For neither Sum nor Difference can be expreſs'd by any Single Root. And from this Addition and Subtraction of Simple Surds only by the Signs, ariſes what they call a *Surd Binomial* or *Reſidual* Root. Thus, $\surd:6+\surd:7$, is a *Binomial Surd*, and $\surd:7-:\surd6$ is a *Reſidual Surd.*

But from *Prop.* 4 *and* 7 *of Euclid's* ſ*econd Book*, there ariſes a *Rule* which helps us to find the *Sum* or *Difference* of *Incommenſurable Square Roots* :

Which Rule is this.

To or from the Sum of the Squares of the given Surd Roots, Add or Subtract their double Rectangle, and the Square Root of the Sum or Remainder, is the Sum or Difference ſought.

Of

Compound Surds.

1. This Binomial $\sqrt[2]{:\overline{u+y}}$ Multiply'd by its corresponding Residual $\sqrt[2]{:\overline{u-y}}$ $=\sqrt[2]{\overline{uu-yy}}$

2. A Table of Powers for Involution in Binomials and Residuals

1	$a \pm b$
2	$a^2 \pm 2ab + b^2$
3	$a^3 \pm 3a^2b + 3ab^2 \pm b^3$
4	$a^4 \pm 4a^3b + 6a^2b^2 \pm 4ab^3 + b^4$
5	$a^5 \pm 5a^4b + 10a^3b^2 \pm 10a^2b^3 + 5ab^4 \pm b^5$
6	$a^6 \pm 6a^5b + 15a^4b^2 \pm 20a^3b^3 + 15a^2b^4 \pm 6ab^5 + b^6$
7	$a^7 \pm 7a^6b + 21a^5b^2 \pm 35a^4b^3 + 35a^3b^4 \pm 21a^2b^5 + 7ab^6 + b^7$
8	$a^8 \pm 8a^7b + 28a^6b^2 \pm 56a^5b^3 + 70a^4b^4 \pm 56a^3b^5 + 28a^2b^6$ [illegible]

E. gr. To find the *Sum* and *Difference* of $\sqrt{:14}$ and $\sqrt{:12}$, their Squares being 14 and 12, their Sum will be 26, and the *Double Rectangle* of $\sqrt{:14}$ into $\sqrt{:12}$, is $2\sqrt{:168}$.

Wherefore $\sqrt{:26 \pm 2\sqrt{:168}}$, is the $\left\{\begin{matrix}\text{Sum}\\ \text{Difference}\end{matrix}\right\}$ required.

Of Compound Surds.

THE Arithmetick of *Compound Surds* depends on the Rules above given about *Simple Surds*, and on the true knowledge of the Signs and + and — in Algebraick Addition, Subtraction, Multiplication and Diviſion; only ſome particular Directions may be given as to Binomials and Reſiduals: As,

I. If any *Binomial* be to be Multiplied by its correſponding Reſidual, the Difference of their Squares is the true Product; and therefore will come out a Rational Quantity, as if $\sqrt{:a+e}$, be Multiplied by $\sqrt{:a-e}$, the Product will be a Rational Quantity, *viz.* $aa - ee$.

II. Involution in Binomials and Reſiduals, is beſt and moſt eaſily performed by a Table of Powers: As becauſe we ſee that $aa + 2ae + ee = \square\,\overline{a+e}$. We may conclude that to Square any Binomial whatſoever; you need only add the double Rectangle of the Parts to the Sum of the Squares of thoſe Parts; or take the double Rectangle from that Sum, if it be a Reſidual.

III. For Diviſion in *Compound Surds*, 'tis convenient, if not neceſſary, to reduce them firſt to ſome better, and when it can be done, to a Rational Form. And

(1.) If a Binomial conſiſting of two Simple Square Roots, or of one Square Root and Rational Quantity, be Multiplied by its correſponding Reſidual, the Product will always be a Rational Quantity.

(2.) If a Binomial conſiſting of two Biquadratick Simple Roots, or of one *ſuch*, and a Rational Quantity; if this be Multiplied by its correſponding Reſidual, the Product will be a Reſidual conſiſting of either two Square Roots, or elſe of one Square Root a Rational Quantity, which Reſidual being Multiplied, as is before ſaid, by its Binomial Produces a Rational Quantity.

(3.) If a Trinomial having three Simple Square Roots, be Multiplied by it ſelf, with one of the Signs changed; the Product will be either a Binomial or Reſidual, which being Muitiplied by its correſpondent Reſidual or Binomial, will give in the Product, a Quantity entirely Rational.

IV. If a Binomial or Reſidual, conſiſting of two Simple Cubick or Biquadratick Roots, &c. or of one

one Cubick or Biquadratick Root, *&c.* and a Rational Quantity is propoſed for a Diviſor; find ſo many continual Proportionals in the Proportion of the Parts of the Binomial or Reſidual propoſed, as there be Units in the Index of the Radical Sign, and ſuch whoſe Radical Sign may be the ſame with that of the Parts of the Binomial or Reſidual; but conjoyned in the Binomial by +, and in Proportionals by + and — alternately; or contrarily, in the Proportionals by +, and in the Reſidual by + and —; the Product of the ſaid Proportionals ſo connexed Multiplied into the Binomial or Reſidual, will be a Quantity entirely Rational. After the ſame manner may a Binomial or Reſidual, having 5 or 6, *&c.* for the Index of the common Radical Sign of the Roots, be reduced to a Quantity entirely Rational.

And Note, *That when the Roots are of different kinds, they muſt firſt be reduced to a common Radical Sign.*

V. If the Diviſor be a Simple Quantity, Divide each part of the Dividend by the Diviſor, and connect thoſe particular Products together by their Signs; but if the Diviſor be a Binomial, Trinomial, or Quadrinomial, *&c.* of ſuch kind as before is ſpecified, reduce that given Diviſor to a new Diviſor that may be a Simple Rational Quantity. Reduce alſo the given Dividend to a new Dividend, by Multiplying the former by the Quantities that were Multiplicators, in reducing the given Diviſor to a Rational Quantity; then Divide the new Dividend by the new Diviſor: But when the Diviſor cannot be reduced to a

Simple Rational Quantity, ſet the Dividend as a Numerator, over the Diviſor as a Denominator.

Thus, $12 + \surd:63$ Divided by 3, the Quotient is $4 + \surd:7$; and $8 - \surd:12$ Divided by 2, the Quotient is $4 - \surd:3$; $\surd:21 + \surd:15$ Divided by $\surd:3$, the Quotient is $\surd:7 + \surd:5$; $\surd:56 + \surd:24$ Divided by $\surd:6$, the Quotient is $\surd:9\tfrac{2}{8} + 2$: and $\sqrt[3]{}:28 - \sqrt[3]{}:14$ Divided by $\sqrt[3]{}:7$, the Quotient is $\sqrt[3]{}:4 - \sqrt[3]{}:2$.

Of

FLUXIONS.

OF

FLUXIONS.

THERE being nothing Publiſh'd on this Subject in our own Language, and yet the vaſt uſe of this *Method of Inveſtigation*, being as conſpicuous, as it is wonderful: I thought it proper to give a ſhort account of it here.

By the *Doctrine of Fluxions*, then we are to underſtand the Arithmetick of the *Infinitely ſmall* Increments or Decrements of *Indeterminate* or *Variable Quantities*, or as ſome call them the *Moments*, or *Infinitely ſmall Differences*, of ſuch Variable Quantities. Theſe Infinitely ſmall Increments or Decrements, our Incomparable Mr. *Iſaac Newton* calls very properly by this Name of *Fluxions*: For, as Indeterminate and Variable Quantities, *viz*. ſuch as in the Generation of Curvilineal and other Figures, by Local Motion, are continually Increaſing or Diminiſhing, he rightly Denominates, *Flowing Quantities*; as being ſuch as are perpetually augmented or leſſen'd by the Flux or Motion of a *Line*, *Surface*, &c. So he calls the *Celerity* or *Velocity* of the Augmentation or Diminution of theſe *Flowing Quantities*, by the Name of *Fluxions*. And becauſe all Figures may be conceived to be generated by Local Motion ; as is now very commonly ſuppoſed among Geometers : Therefore 'tis much more Natural to conceive the Infinitely ſmall Increments or Decrements of the Variable and *Flowing Quantities*, under the Notion of *Fluxions*, than under that of *Moments* or Infinitely ſmall *Differences* ; as *Leibnits*, *Niewentiit*, and the Noble Author of *Analyſe des Infiniment Petits* chuſe rather to take them : Tho' even that way alſo is not without its Uſe in many Caſes.

The Excellent Mr. *Newton* ſuppoſes the *Abſciſſa* of a Curve, or any other Flowing or Variable Quantity to be uniformly augmented, and therefore for its *Fluxion* he puts 1, or Unity. And the other Flowing Quantities he denotes uſually by the Letters v, x, y, z, and expreſſes their *Fluxions* by only repeating the ſame Letters with Points or Pricks over their Heads ; thus, $\dot{v}$, $\dot{x}$, $\dot{y}$, $\dot{z}$: Which are the Fluxions of the former Flowing Quantities. And this Method is much more natural and ſhorter than *Niewentiit*'s, or the *French* one with the Differential *d* Multiplied into the Flowing Quantity, to denote the Fluxion.

And becauſe theſe *Fluxions* themſelves are alſo Indeterminate and Variable Quantities, and do continually Increaſe or Decreaſe, or grow greater or leſſer. Therefore he conſiders the Velocities with which they do ſo Increaſe or Diminiſh, as the *Fluxions* of the former *Fluxions* : And thoſe may be called *Second Fluxions*, and are noted with two Points over them, thus ; $\ddot{y}$, $\ddot{x}$, $\ddot{z}$. And if you go on again, and conſider the perpetual Augmentation or Diminution of *theſe*, as their *Fluxions* alſo,

alſo, you may make third, fourth, or fifth *Fluxions*, &c. which will be noted thus; $\overset{\therefore}{y}, \overset{\therefore}{x}, \overset{\therefore}{z}$; $\overset{::}{y}, \overset{::}{x}, \overset{::}{z}$,; $\overset{:\cdot:}{y}, \overset{:\cdot:}{x}, \overset{:\cdot:}{z}$: And ſo on *ad Infinitum.* If the Flowing Quantity be a Surd or a Fraction, he thus expreſſes its *Fluxion*: Let the Surd be $\sqrt{}:\overline{a-b}$, its Fluxion is $\sqrt{}:\overset{\cdot}{\overline{a \dot{-} b}}$: And the Fluxion of the Fraction $\frac{xx}{d-y}$ is $\overset{\cdot}{\underset{\cdot}{\frac{xx}{d-y}}}$. See Dr. *Wallis*'s *Latin Edit. Pag.* 392.

The main Buſineſs of the *Algorithm*, or *Arithmetick of Fluxions*, conſiſts in theſe two Things.

I. *From the Flowing Quantity given, to find the Fluxion.*

II. *From the Fluxion, to find the Flowing Quantity.*

As to the former of theſe, the Learned Dr. *Wallis*, in the place above mentioned, (from Mr. *Newton*'s Papers) gives this General Rule.

Let each Term of the Equation be Multiplied ſeparately by the ſeveral Indexes of the Powers of all the Flowing Quantities contain'd in that Term: And in every ſuch Multiplication let one Root or Letter of the Power be changed into its proper Fluxion: So ſhall the Aggregate of all the Products connected together by their proper Signs, be the Fluxion of the Equation deſired.

And all the Caſes of it are demonſtrated by Mr. *Iſaac Newton* in the *Lemmata* above delivered, which I ſhall exemplifie by particular Inſtances.

I. In the General, to expreſs the Fluxions of Simple Variable Quantities, as was ſaid before, you need only uſe the Letter or Letters which expreſs them, with a ſmall Point over their Heads, thus. The Fluxion of x is $\dot{x}$, and the Fluxion of y is $\dot{y}$, and the Fluxion of $x + y + v + z$, is $\dot{x} + \dot{y} + \dot{v} + \dot{z}$, &c.

And (Inverſely) the *Flowing Quantities* in this caſe will be eaſily had from the Fluxions, by only writing the Letters without the Points over them.

N. B. *For the Fluxion of Permanent Quantities, when any ſuch are in the Equation, you muſt imagine* o *or a Cypher; for ſuch Quantities can have no Fluxions, properly ſpeaking, becauſe they are without Motion, or Invariable.*

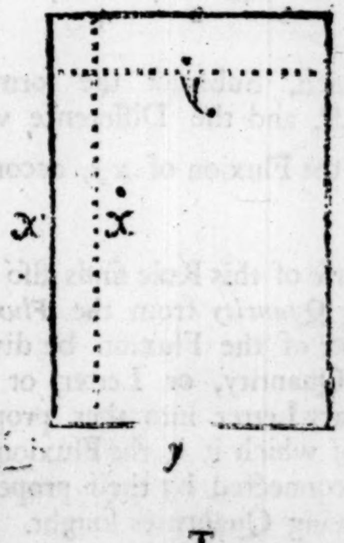
II. To find the Fluxions of the Products of two or more Variable or Flowing Quantities: *Multiply the Fluxion of each Simple Quantity, by the Factors of the Products, or the Product of all the reſt, and connect the laſt Products by their proper Signs, the Sum, or Aggregate, is the Fluxion ſought.*

Thus

Thus the Fluxion of xy is $x\dot{y} + \dot{y}x$, and the Fluxion of xyz is $xy\dot{z} + x\dot{y}z + \dot{x}yz$; and the Fluxion of $xvyz$, is $xvy\dot{z} + xv\dot{y}z + x\dot{v}yz + \dot{x}vyz$, and the Fluxion of $\overline{a+x} \times$ by $b-y$ (the common Product being $ab + bx - ya - xy$) will be $b\dot{x} - \dot{y}a - \dot{x}y - x\dot{y}$.

Demonſtration of Rule 2.

Suppoſe xy = to any Rectangle made or encreaſed by a Perpetual Motion or Fluxions of either of the Sides x or y, along the other, and let the Moments, or Fluxions of the Sides, be $\dot{x}$ and $\dot{y}$. By which we underſtand the *Velocity* with which either Side moves to form the Rectangle.



From the Sides ſubduct the half Fluxions of thoſe Sides, and it will ſtand thus; $x - \frac{\dot{x}}{2}$ and $y - \frac{\dot{y}}{2}$.

Multiply theſe one into another; and the Product will be $x\,y - \frac{y\dot{x}}{2} - \frac{x\dot{y}}{2} + \frac{\dot{x}\,\dot{y}}{4}$.

Then to the Sides add the half Moments, or Fluxions, and it will be thus: $x + \frac{\dot{x}}{2}$, and $y + \frac{\dot{y}}{2}$: Which Multiplied alſo into one another, will produce $x\,y + \frac{y\dot{x}}{2} + \frac{x\dot{y}}{2} + \frac{\dot{x}\,\dot{y}}{4}$.

After which, Subtract the former Product from this laſt, and the Difference will be only $x\,\dot{y} + \dot{x}\,y$, the Fluxion of $x\,y$, according to the Rule.

The Inverſe of this Rule finds alſo (in this caſe) the *Flowing Quantity* from the *Fluxion, viz.* If each Member of the Fluxion be divided by the Fluxionary Quantity, or Letter, or by changing the Fluxionary Letter into that proper Flowing Quantity of which it is the Fluxion. For then the Quotes connected by their proper Signs, will be the Flowing Quantities ſought. Only if the

Letters be all exactly the ſame, the Flowing Quantity will be a Simple one, whoſe parts are not to be connected together by the Signs + and —, as in the firſt three Examples of this Rule.

III. To find the Fluxion of any Fraction. *Multiply the Fluxion of the Numerator by the Denominator, and after it place (with the Sign —) the Fluxion of the Denominator into the Numerator; and Divide the whole by the Square of the Denominator.*

Thus the Fluxion of $\frac{x}{y}$ is $\frac{\dot{x}y - x\dot{y}}{yy}$, for ſuppoſe $\frac{x}{y} = z$: Then will $x = yz$; which equal Quantities will have equal Fluxions; therefore $\dot{x} = \dot{y}z + \dot{z}y$, and $\dot{x} - z\dot{y} = \dot{z}y$; and dividing all by y: $\frac{\dot{x} - z\dot{y}}{y} = \dot{z} = \left(\text{becauſe } \frac{x}{y} = z\right)$ $\frac{y\dot{x} - x\dot{y}}{yy}$ wherefore this laſt is the Fluxion of the Fraction $\frac{x}{y} = z$; becauſe z being $= \frac{x}{y}$, $\dot{z}$ will will be equal to the Fluxion of $\frac{x}{y}$. And the Fluxion of $\frac{a}{x}$ will be $\frac{-\dot{x}a}{xx}$; for the Permanent Quantity a, having no Fluxion, there can be no Product of the Fluxion of the Numerator into the Denominator, as there would have been, had a been x, z, or any other Variable Quantity.

Alſo the Fluxion of $\frac{x}{a}$ will be $\frac{\dot{x}a}{aa}$; and the Fluxion of $\frac{x}{a+x}$, will be $\frac{\dot{x}a + \dot{x}x - x\dot{x}}{aa + 2ax + xx}$, or, $\frac{\dot{x}a}{aa + 2ax + xx}$; becauſe $- \dot{x}x$ and $+ \dot{x}x$ deſtroy one another: Alſo the Fluxion of $\frac{1}{x}$, or, $x - 1$, is $\left(\frac{-\dot{x}}{x^2}.\right)$ Here alſo the Reverſe of the Rule ſerves to find the Flowing Quantity from the Fluxion; as if the Flowing Quantity of this Fraction were required $\frac{\dot{x}y - x\dot{y}}{yy}$. Firſt, Multiply it by the Square of the Denominator, and it will be $\dot{x}y - x\dot{y}$, from which take away $-x\dot{y}$, which was placed after it, and it will be $\dot{x}y$; omit the Point, and 'tis xy, which becauſe y is the Denominator of a Fraction, will at laſt be $\frac{x}{y}$.

Before the Fluxions of any Power, whether Perfect or Imperfect, can be found; the following way of Notation muſt be well underſtood.

If a Series of Geometrick Progreſſionals be in this Order,

$1.\ x.\ xx.\ xxx.\ x^4.\ x^5.\ x^6.\ x^7$, &c.

The

Their Indexes, or Exponents, will be in Arithmetical Progreſſion, and ſtand thus.

0. 1. 2. 3. 4. 5. 6. 7.

But if they are Fractions; as,

$$\frac{1}{x} \quad \frac{1}{xx} \quad \frac{1}{x^3} \quad \frac{1}{x^4} \quad \frac{1}{x^5} \quad \frac{1}{x^6} \quad \frac{1}{x^7}$$

Then their Exponents will be Negative; and ſtand thus.

$-1.\ -2.\ -3.\ -4.\ -5.\ -6.\ -7.$

For if you ſuppoſe $x = 2$: Then will $\frac{1}{x} = \frac{1}{2}$, and $\frac{1}{xx} = \frac{1}{4}$, and $\frac{1}{xxx} = \frac{1}{8}$, &c.

Or if you expreſs the *Geometrical Series* by means of the Exponents, it will ſtand thus, x^{-1} x^{-2}, &c. And if it were expreſſed thus x^0; then it will be $x^0 = 1$, becauſe x is the Denominator of the *Ratio*, in which Unity is not affected. Thus alſo, $\frac{1}{x^4} = x^{-4}$ and $\frac{1}{x^3} = x^{-3}$. And $1 = x^0$, $x^1 = x$, $x^2 = xx$, &c.

Alſo the Exponent of $\surd : x$ will be $\frac{1}{2}$, becauſe as $\surd : x$ is a Mean Proportional between 1 and x. So $\frac{1}{2}$ is an Arithmetical Mean between 0 and 1.

And the Exponent of $\sqrt[3]{\ }: x$ will be $\frac{1}{3}$; becauſe as $\sqrt[3]{\ }: x$ is the firſt of the two Mean Proportionals between 1 and x; ſo $\frac{1}{3}$ is the firſt of the two Arithmetical Means between 0 and 1.

For ſince $1\ \overset{2}{x}.\ x\,\overset{4}{x}.\ x\,x\,\overset{8}{x}.$ are continually proportional; therefore their Cubes, or any other Roots, will be alſo continually proportional. That is, $\sqrt[3]{\ }: 1\ (=1.)\ \sqrt[3]{\ }x.\ \sqrt[3]{\ }x\,x.\ \sqrt[3]{\ }x\,x\,x\,(=x)$ ∺

So alſo,

$$1.\ x,\ x^2.\ x\,x\,x.\ x^4.\ x^5.\ ∺$$

Wherefore the Roots of the Fifth Power of thoſe Quantities will be ∺

That is,

$$\sqrt[5]{\ }1.\ \sqrt[5]{\ }x.\ \sqrt[5]{\ }x^2.\ \sqrt[5]{\ }x^3.\ \sqrt[8]{\ }x^4.\ \sqrt[5]{\ }x^5.\ (=x.)$$

Alſo, for the ſame Reaſon, the Exponent of $\sqrt[5]{\ }: x^4$, will be $\frac{4}{5}$.

N. B. Always place the Index of the Letter (or Power) over that of the Radical Sign.

Thus

Thus in Fractions: The Exponent of $\frac{1}{x}$ will be — 1, of $\frac{1}{\sqrt{:x^3}}$ will be — $\frac{3}{2}$, of $\frac{1}{\sqrt[3]{:x^5}}$ will be — $\frac{5}{3}$, of $\frac{1}{\sqrt{:x^7}}$, will be — $\frac{7}{2}$, &c.

N. B. $\sqrt{:x}$ and $x^{\frac{1}{2}}$, or $\sqrt[3]{\cdot x}$ and $x^{\frac{1}{3}}$: Or, $\sqrt[5]{:x^4}$, and $x^{\frac{4}{5}}$, are only two different ways of Notation for one and the ſame thing, the former is the *old*, the latter in the *new* Way.

So likewiſe $\frac{1}{x}$ and x^{-1} are all one: And $\frac{3}{x}$ is x^{-3}, &c.

The way of reading or expreſſing Quantities ſo denoted, is thus, x^{-3} is Unity divided by the Cube of x, and if it were $x^{-\frac{7}{3}}$; it muſt be read, Unity or One, divided by the Cube-Root of the Seventh Power of x.

Note alſo, That the Sum of any two Exponents of two Numbers or Quantities, in Geometrick Progreſſion, makes the Exponent of the Product of thoſe two Terms.

Thus $x^{\frac{1}{2}+\frac{1}{3}}$ or $x^{\frac{5}{6}}$ is the way of expreſſing the Product of $x^{\frac{1}{3}}$ into $x^{\frac{1}{2}}$, and $x^{-\frac{1}{5}+\frac{4}{5}}$, or $x^{-\frac{2}{5}}$ is the Product of $x^{-\frac{1}{5}}$ into $x^{\frac{4}{5}}$.

Alſo $x^{-\frac{1}{3}-\frac{1}{3}}$, or $x^{-\frac{2}{3}}$ is the Product of $x^{-\frac{1}{3}}$ into it ſelf, or the Square of $x^{-\frac{1}{3}}$.

And the Difference between the Exponents of any two Terms, is the Exponent of the Quotient ariſing by Diviſion of the greater by the leſs.

Thus, $x^{\frac{1}{2}-\frac{1}{3}}$ —, or $x^{\frac{1}{6}}$ is the Exponent of the Quotient of $x^{\frac{1}{2}}$ by $x^{\frac{1}{3}}$, &c.

Let p repreſent the Exponent of N, any Number at pleaſure; and let $p = 1$.

Then will $N^{p} = N^{1} : N^{p+1} = N^{2}$, and $N^{p+2} = N^{3} : N^{p+3} = N^{4}$, &c.

Or, if $p = 3$.

Then will $N^{p} = N^{3}$ and $N^{p+3} = N^{6}$, &c.

And Negatively,

$N^{-p} = N^{-3}$, and $N^{-p+3} = N^{0}$, &c.

Alſo, 0 is an Arithmetical Mean between a Poſitive and a Negative Quantity equally diſtant from it, (*i. e.*) —6 : 0 : 6, are Arithmetically proportional: So is 1 a Geometrical Mean between an Affirmative and Negative Power, at equal Diſtances from it.

That is, $N^{-p} : 1 : N^{p}$ ∺

Wherefore, $1 = N^{-p} \times N^{p}$. And dividing all by N^{p}: $\frac{1}{N^{p}} = N^{-p}$.

And to add ſome Examples of Multiplication and Diviſion in this way:

$$\frac{1}{x} \times \frac{1}{\sqrt[3]{x^{5}}}. = x^{-1} \times x^{\frac{5}{3}} = -\frac{1}{3} \times x^{\frac{5}{3}} = x^{-\frac{8}{3}}$$

$$= \frac{-1}{x^{\frac{8}{3}}} = \frac{1}{\sqrt[3]{x^{8}}},$$ &c.

And $\frac{1}{\sqrt[3]{x^{5}}}$, divided by $\frac{1}{x}$, will ſtand in this Notation thus, $\frac{1}{x}\Big)\frac{1}{\sqrt[3]{x^{5}}}, (= x^{-1})\, x^{-\frac{5}{3}} (= x^{-\frac{1}{3}})$

$$x^{-\frac{5}{3}} = (^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{xx}},$$ &c.

IV. This being well underſtood, there is this general Rule for the finding the Fluxion of any Power, whether Perfect or Imperfect, *viz. Multiply the Power (firſt brought one Degree lower) by the Index of that firſt Power; and then, that Product by the Fluxion of the Root.*

Thus the Fluxion of $x x$ will be $2 x \dot{x}$, ſor $x x = x \times x$, but the Fluxion of $x \times x = \dot{x} x + x \dot{x} = 2 x \dot{x}$, &c. and the Fluxion of x^3 will be $3 x x \dot{x}$: That of x^{8} will be $8 x^7 \dot{x}$, &c. or if m expreſs the Index of any Power, as ſuppoſe x^m. Then its Fluxion will be $m \dot{x}^{m+1} \dot{x}$ or $m \dot{x} x^{m-1}$. For x^m brought one degree lower (m being a general Index) muſt be x^{m-1}: Then that Multiplied by m the Index, makes $m x^{m-1}$, and this laſt by the Fluxion of the Root produces $m x^{m-1} \dot{x}$.

If the Power be produced from a Binomial, *&c.* as ſuppoſe $x x + 2 x y + y y$, its Fluxion will be $2 x \dot{x} + 2 \dot{x} y + 2 x \dot{y} + 2 y \dot{y}$, by working according to this fourth, and the ſecond Rules.

If the Exponent be Negative; as ſuppoſe x^{-m} or $\frac{1}{x^m}$, its Fluxion will be $- m \dot{x} x^{-m-1}$; or if you would do it by way of Fraction $\frac{-m x^{m-1} \dot{x}}{x^{2m}}$ (for the Square of x^m is as well x^{2m} as x^{m2}) or, according to Mr. *Newton*'s way, which is yet ſhorter, $\frac{-m x}{x^{m+1}}$. See *Caſe* 4. Page 252 of his *Principia.*

If the Power be imperfect, *i. e.* if its Exponent be a Fraction, as suppose $\sqrt[n]{:x^{m}}$: Or in the other Notation $x^{\frac{m}{n}}$. Let us suppose $x^{\frac{m}{n}} = z$. Then if you raise up each Member to the Power of x, it will stand thus $x^{m} = z^{n}$. The Fluxion of which will be, by this general Rule, $m x^{m-1} \dot{x} = n z^{n-1} \dot{z}$. Wherefore $\dot{z}$ will be $= \frac{m \dot{x} x^{m-1}}{n z^{n-1}}$; (by dividing both Parts by $n z^{n-1}$): And $\frac{m \dot{x} x^{m-1}}{n z^{n-1}} = \frac{m}{n} x^{\frac{m-n}{n}} \dot{x}$: or $\frac{m}{n} \dot{x} \sqrt[n]{: x^{m-n}}$, by putting instead of $n z^{m+1}$ is Value $n x^{m-\frac{m}{n}}$, So that to find the Fluxion of any kind of Power you must proceed thus,

Multiply the Power given by its Index or Exponent, and then that Product by the Fluxion of the Root of the Power given, and after this Subduct one or Unity from the Index of the Power.

As for the Fluxions of Surd Quantities, Mr. *Hayes* gives you many Examples in his Treatise of Fluxions, lately published, which will make the thing plain to any one that will render himself ready at the Practice of this Art. See his Book, *Prop.* VII. *pag.* 14.

Next for the Rule *Inverſly*, to find the Flowing Quantity belonging to the Fluxion of any Power, whether Perfect or Imperfect; proceed thus,

I. Take the Fluxionary Letter, or Letters, out of the Equation.

II. Augment the Index of the Fluxion by 1, or Unity.

III. Divide the Fluxion by the Index of its Power ſo iucreaſed by Unity.

Examples.

If $3\,x\,x\,\dot{x}$ were propoſed: by taking away $\dot{x}$, it will be $3\,x\,x$; and by encreaſing its Index by Unity, it will be $3\,x\,x\,x$. Then dividing it by it 3, its now (augmented) Index, the Quotient will be $x\,x\,x$, the Flowing Quantity required.

Again:

Suppoſe $\frac{n}{m}\,\dot{x}\,x^{\frac{n}{m}-1}$ a Fluxion propoſed: By taking away the Fluxionary x, it will be $\frac{n}{m}\,x^{\frac{n}{m}-1}$: By augmenting the Index by Unity, (*i. e.*) taking away — 1. it will be $\frac{n}{m}\,\dot{x}^{\frac{n}{m}}$: And laſtly, by dividing the remaining part of the Fluxion by $\frac{n}{m}$ prefixed to, or Multiplied into x, the Quotient will be $x^{\frac{n}{m}}$: Which is the Flowing Quantity ſought.

You will find Examples enough of this *Inverſe Method, viz.* The *Calculas Integralis*, or *Summatory Arithmetick*, in Mr. *Hayes*'s Book of Fluxions, *Sect.* 4.

The deſigned brevity of this Tract will permit me to give you only two *Uſes* of this Doctrine of Fluxions, which, I hope, may ſerve to give the Inquiſitive Reader a whet for a further perſuit of this matter; and he will find ſufficient Satisfaction, by peruſing the Authors above-mentioned, *viz* *Newton*, *Wallis*, *Niewentiit*, *Carre*, *Leibnitz*, (in the *Act. Eruditor. Lipſiæ*) and eſpecially the Maquis *L'Hoſpital*, his excellent *Analyſe des Infiniment Petits*: Conſult alſo the Ingenious Mr. *Abraham de Moivre*, *Specimina Doctrinæ Fluxionum* in *Philoſoph. Tranſact.* N. 216. where you have much in a little on this Subject; And the Marrow of moſt of theſe Authors you have in Mr. *Haye*'s Treatiſe of Fluxions.

I. To find the Area of a Parabola.

Let the Parameter be $p = 1$, let $x =$ to the *Abſciſſa*, and $y =$ to the Ordinate.

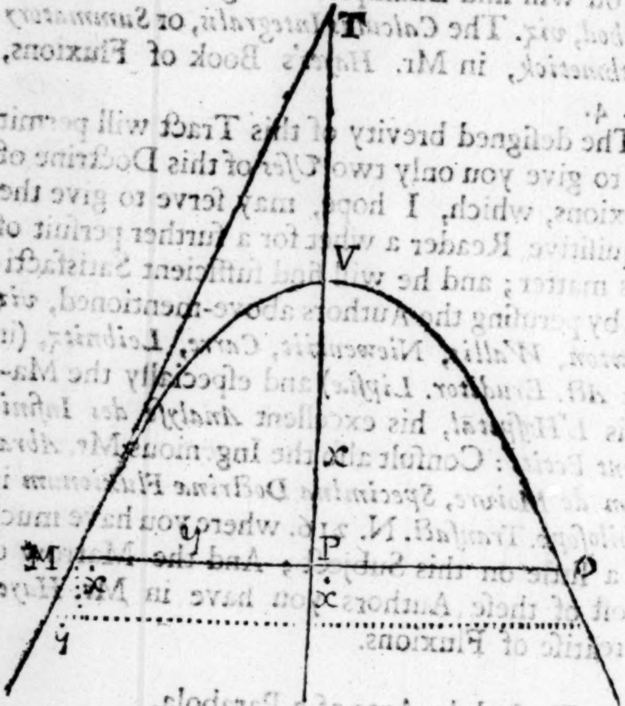
Then by the Property of the *Curve* $x = yy$ (becauſe $p = 1$.)

And conſequently, by Extraction of the Square Roots of each, and uſing the New Notation: $x^{\frac{1}{2}} = y$.

Then Multiplying $x^{\frac{1}{2}}$ by x, the Fluxion of the *Abſciſſa*, it will ſtand thus $\dot{x}\, x^{\frac{1}{2}} =$ to the Fluxion of the Area.

Laſtly, Find the Flowing Quantity anſwering to that Fluxion, and that ſhall give the Area in known Terms.

To



To do which, 1. Take away $\dot{x}$, and it will be $x^{\frac{1}{2}}$.

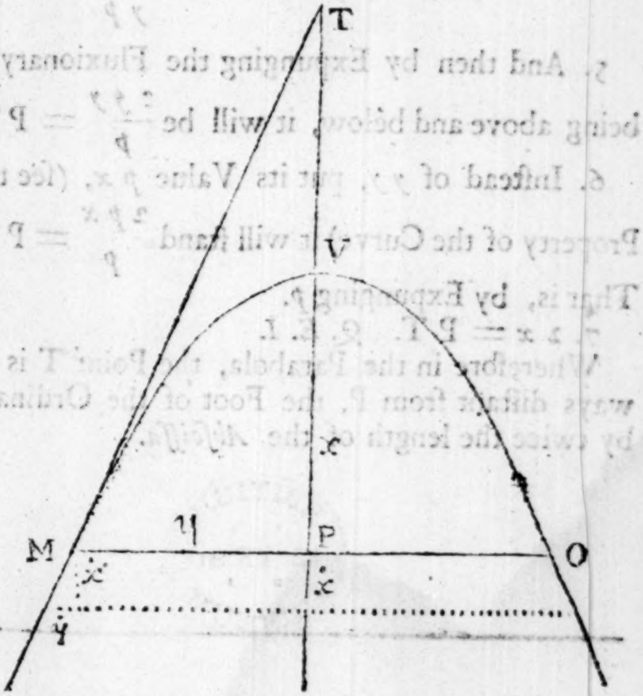
2. Encreaſing the Power of $x^{\frac{1}{2}}$ by Unity, it will ſtand $x x^{\frac{1}{2}}$.

3. Divide $x x^{\frac{1}{2}}$ by $1 + \frac{1}{2}$, or $\frac{3}{2}$; thus, $\frac{3}{2}\Big)\frac{x x^{\frac{1}{2}}}{1}\Big(\frac{2 x x^{\frac{1}{2}}}{3}$: And the Quotient will be $\frac{2 x x^{\frac{1}{2}}}{3}$.

Laſtly,

Laſtly, inſtead of $x^{\frac{1}{2}}$ Subſtitute its Value or equal p: And it will be $\frac{2}{3}xy =$ to the Area m V p, and that doubled, gives the whole Area of the Parabola m V O.

II. To find the Point T, where T M being drawn, ſhall be true Tangant to the Parabola, The Property of the *Curve* is $px = yy$.



1. Find the Fluxion of that Equation, which is $p\dot{x} = 2y\dot{y}$. Wherefore,

2. $\dot{x} = \frac{2y\dot{y}}{p}$.

3. But by Similar Triangles, $\dot{y}, \dot{x} :: y, \frac{\dot{x}y}{\dot{y}} =$ P T the Sub-tangent.

4. Therefore Subſtituting $\frac{2y\dot{y}}{p}$ (which is $= \dot{x}$, by *Step.* 2.) inſtead of $\dot{x}$, it will be $\frac{2yy\dot{y}}{\dot{y}p} =$ P T.

5. And then by Expunging the Fluxionary $\dot{y}$, being above and below, it will be $\frac{2yy}{p} =$ P T.

6. Inſtead of yy, put its Value px, (ſee the Property of the Curve) it will ſtand $\frac{2px}{p} =$ P T. That is, by Expunging p.

7. $2x =$ P T. *Q. E. I.*

Wherefore in the Parabola, the Point T is always diſtant from P, the Foot of the Ordinate, by twice the length of the *Abſciſſa.*

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FINIS.

see the leaf next to page 40th in the backside.

18 $m : n :: \frac{Fm - a}{n} : \frac{Fm - a}{m} = 39745\frac{10}{18}$ the number of steps B has ~~to ye goal~~ run after he's overtaken, when D comes to ye goal

19 $s : r :: \frac{Fm - a}{m} : \frac{Fmr - ra}{s} = 31796\frac{4}{9}$ the number of D's steps that he was before B when he came to ye goal.

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